Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/44-1.2.2.7-P-x-d+e-x^2-^q-a+b-x^2+c-x^4-^p

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [42]. This is test number [44].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (42)	0.00 (0)
Mathematica	95.24 (40)	4.76 (2)
Maple	95.24 (40)	4.76 (2)
Fricas	61.90 (26)	38.10 (16)
Giac	14.29 (6)	85.71 (36)
Sympy	14.29 (6)	85.71 (36)
Mupad	2.38 (1)	97.62 (41)
Maxima	0.00 (0)	100.00 (42)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

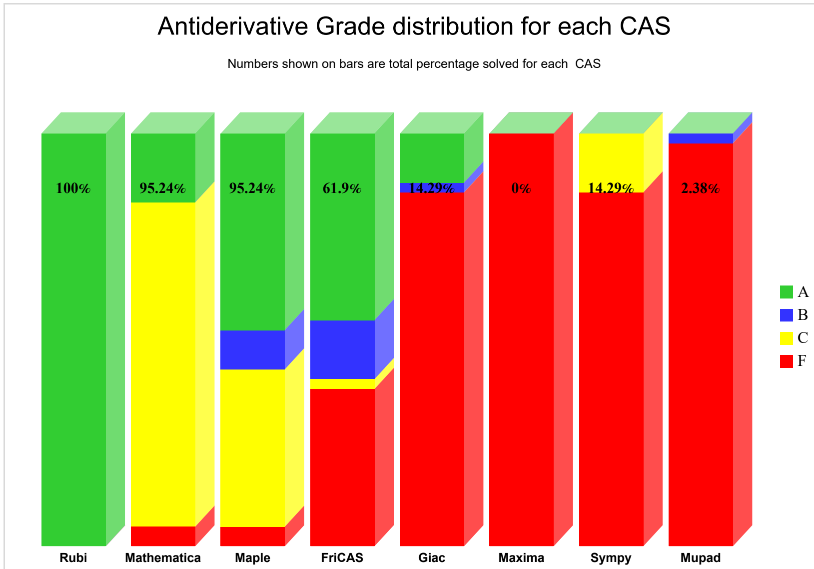
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

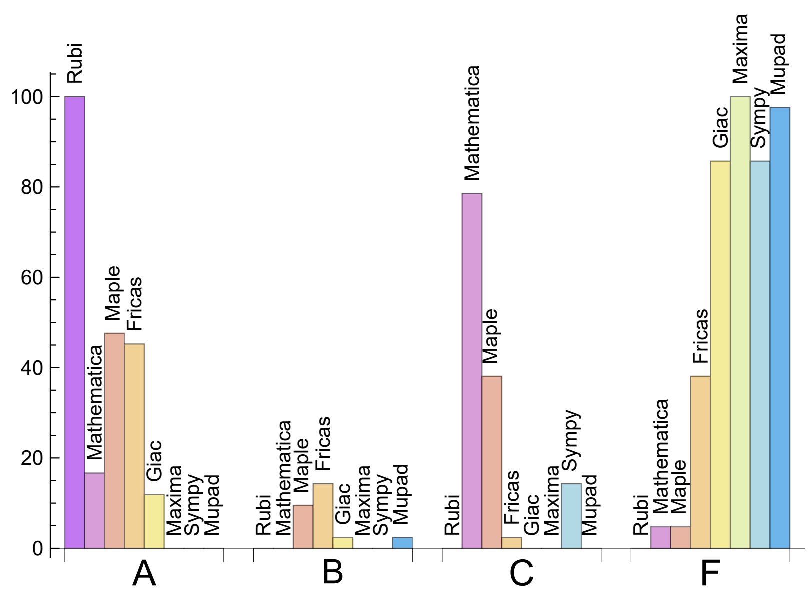
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	47.619	9.524	38.095	4.762
Fricas	45.238	14.286	2.381	38.095
Mathematica	16.667	0.000	78.571	4.762
Giac	11.905	2.381	0.000	85.714
Mupad	0.000	2.381	0.000	97.619
Maxima	0.000	0.000	0.000	100.000
Sympy	0.000	0.000	14.286	85.714

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Fricas	16	31.25	68.75	0.00
Giac	36	97.22	0.00	2.78
Sympy	36	80.56	19.44	0.00
Mupad	41	0.00	100.00	0.00
Maxima	42	83.33	0.00	16.67

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Giac	0.33
Fricas	0.34
Rubi	0.51
Maple	2.58
Sympy	2.61
Mupad	8.63
Mathematica	8.73
Maxima	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	186.00	0.53	167.00	0.51
Giac	330.17	1.20	310.00	1.04
Mupad	397.00	3.75	397.00	3.75
Mathematica	423.88	0.83	259.50	0.68
Rubi	576.24	1.08	396.50	1.00
Fricas	603.31	1.63	458.50	1.33
Maple	863.42	1.26	360.50	0.88
Maxima	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

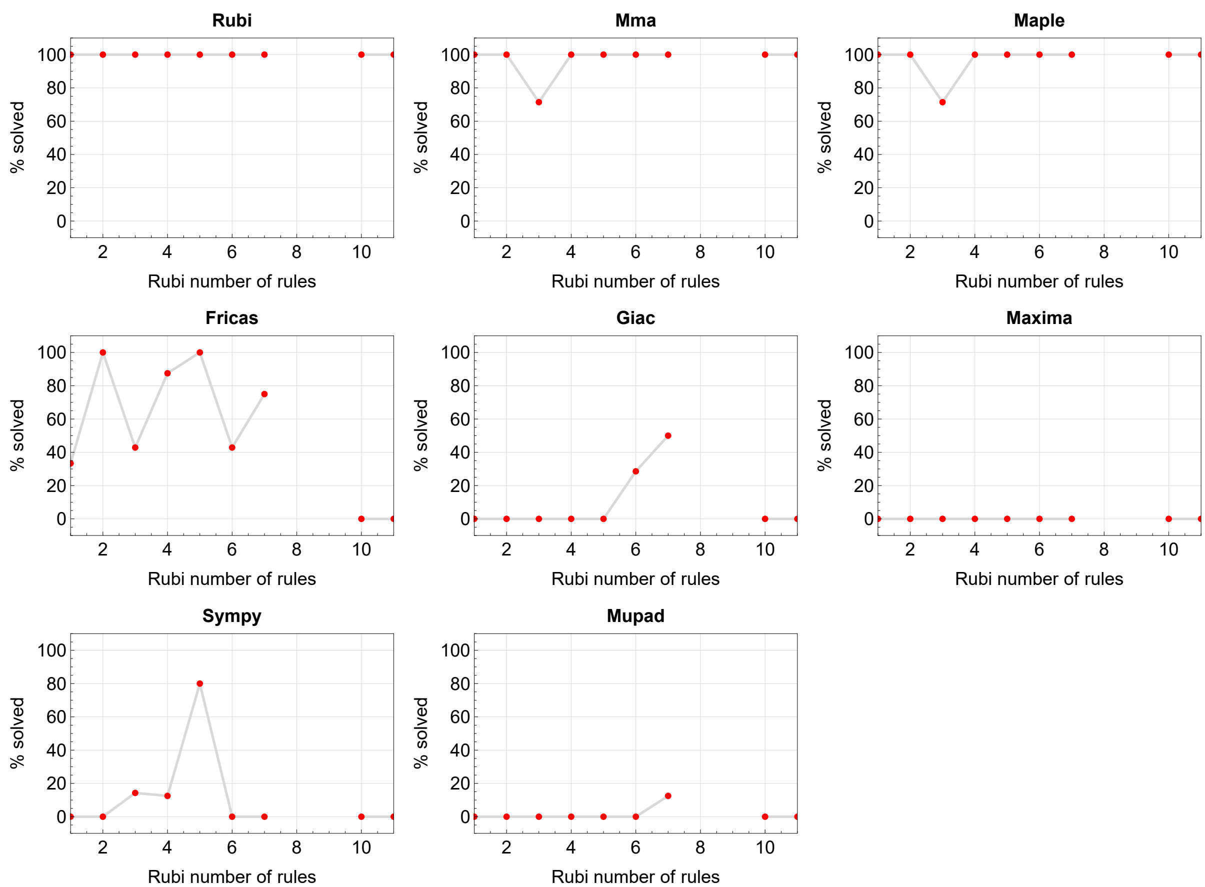


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

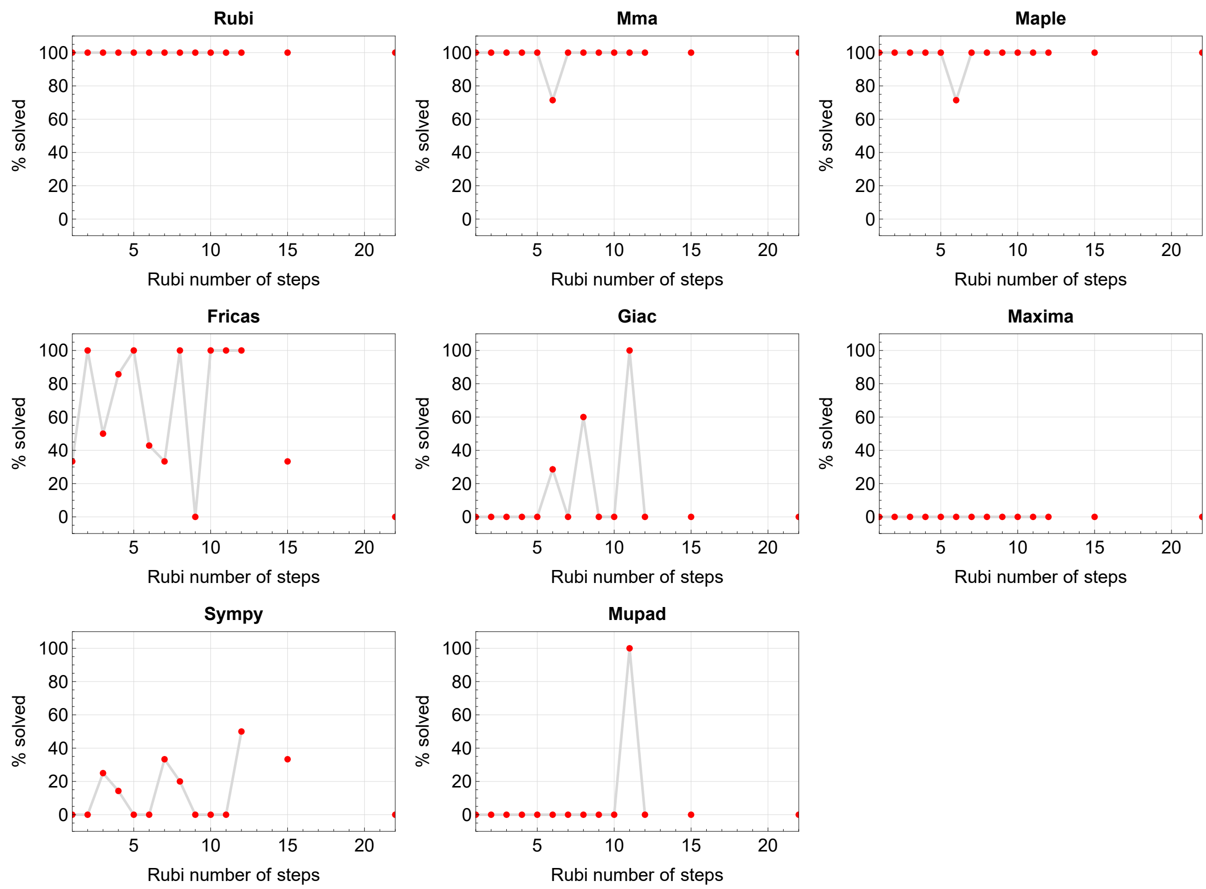


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

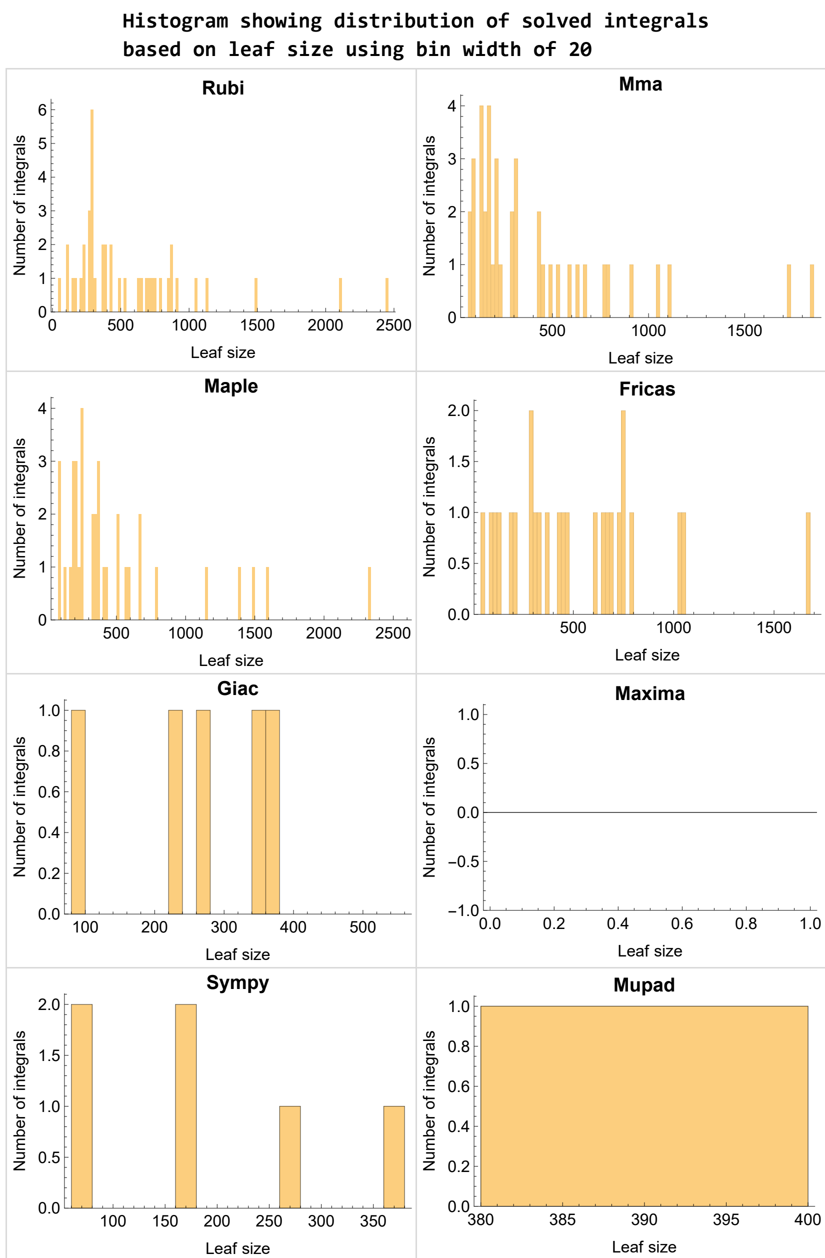


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

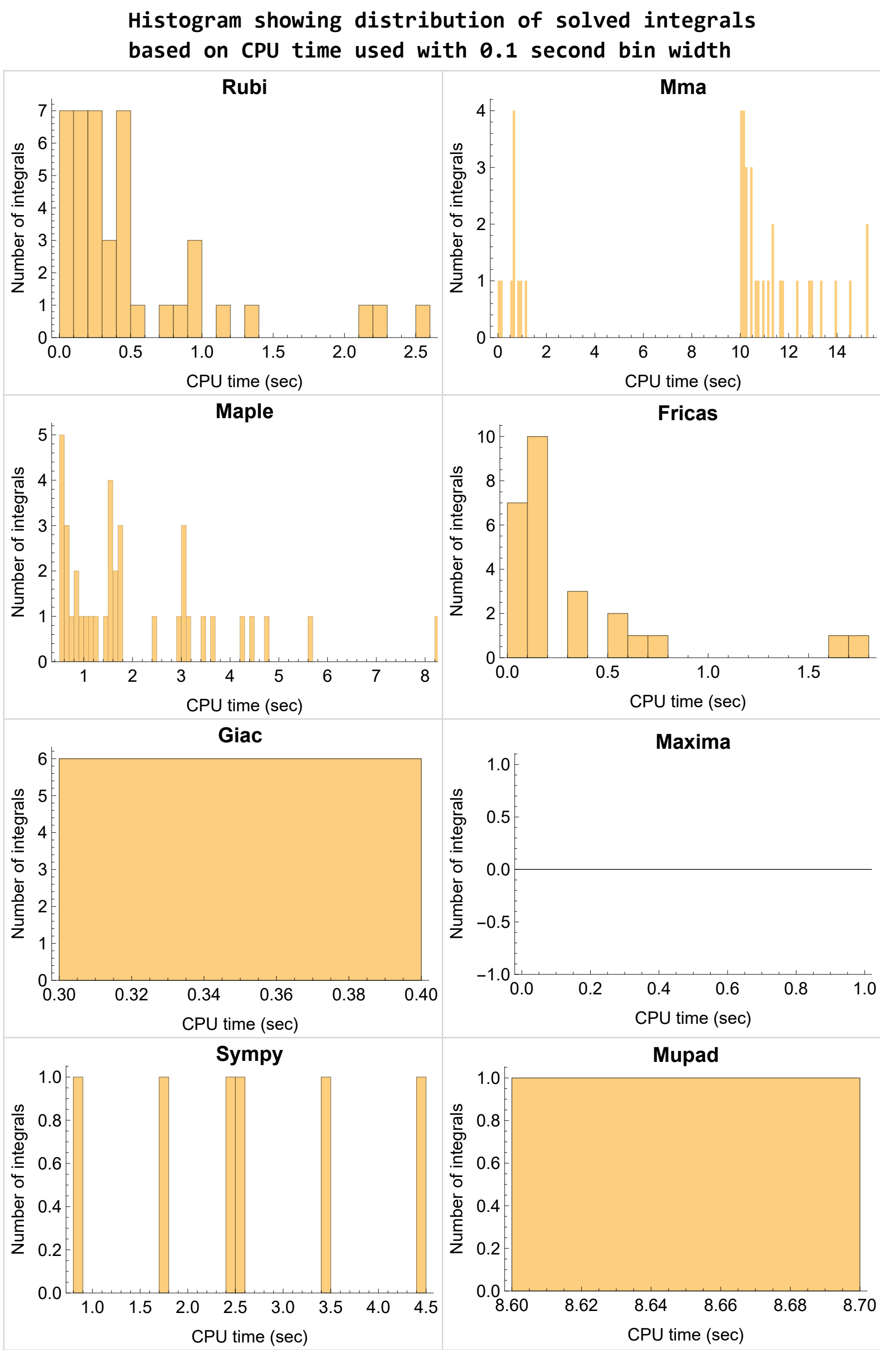


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

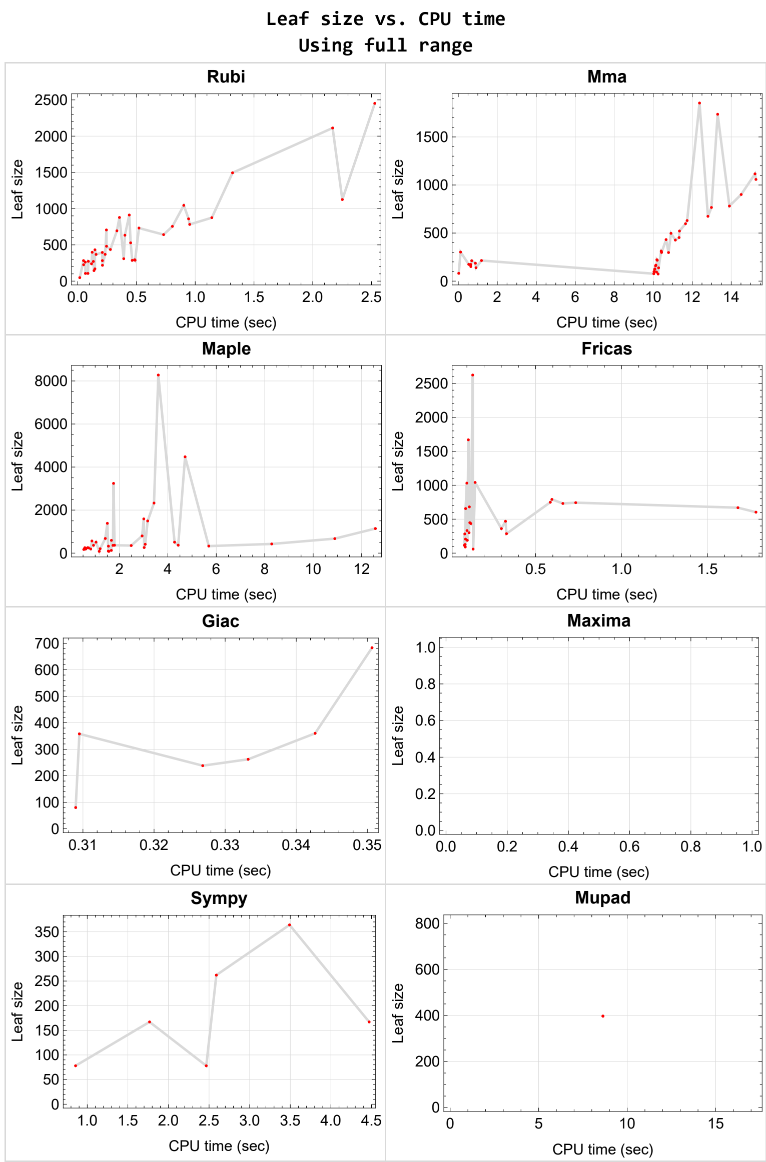


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `sleliervre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

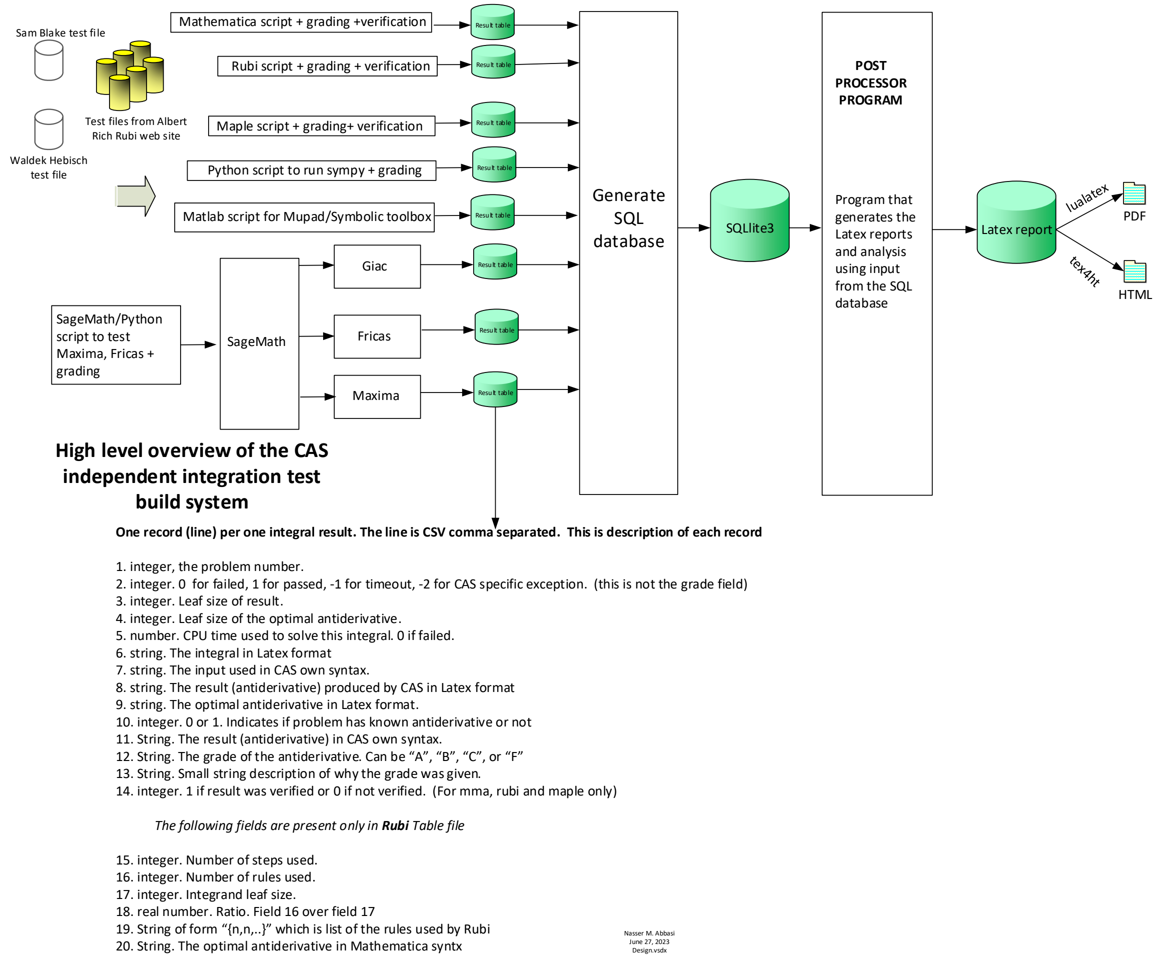
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 35, 37, 38, 39, 40, 41, 42 }

B grade { }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36 }

F normal fail { 15, 33 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 17, 18, 19, 20, 21, 24, 25, 26, 27, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

B grade { 22, 23, 28, 29 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 32 }

F normal fail { 15, 33 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 17, 18, 19, 20, 27, 37, 38, 39, 40, 41, 42 }

B grade { 24, 25, 26, 34, 35, 36 }

C grade { 16 }

F normal fail { 5, 15, 22, 32, 33 }

F(-1) timedout fail { 6, 7, 12, 13, 14, 21, 23, 28, 29, 30, 31 }

F(-2) exception fail { }

Maxima

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36 }

F(-1) timedout fail { }

F(-2) exception fail { 30, 37, 38, 39, 40, 41, 42 }

Giac

A grade { 34, 37, 38, 40, 41 }

B grade { 42 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36 }

F(-1) timedout fail { }

F(-2) exception fail { 39 }

Mupad

A grade { }

B grade { 34 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42 }

F(-2) exception fail { }

Sympy

A grade { }

B grade { }

C grade { 1, 2, 3, 4, 10, 11 }

F normal fail { 5, 6, 7, 8, 9, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40 }

F(-1) timedout fail { 14, 15, 28, 29, 33, 41, 42 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	878	217	327	0	283	364	0	0
N.S.	1	1.94	0.48	0.72	0.00	0.62	0.80	0.00	0.00
time (sec)	N/A	0.355	10.207	5.690	0.000	0.088	3.491	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	706	159	258	0	206	262	0	0
N.S.	1	1.92	0.43	0.70	0.00	0.56	0.71	0.00	0.00
time (sec)	N/A	0.244	10.120	3.023	0.000	0.092	2.590	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	432	121	203	0	127	167	0	0
N.S.	1	1.56	0.44	0.73	0.00	0.46	0.60	0.00	0.00
time (sec)	N/A	0.147	10.074	1.205	0.000	0.090	1.767	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	77	169	0	90	78	0	0
N.S.	1	1.00	0.34	0.75	0.00	0.40	0.35	0.00	0.00
time (sec)	N/A	0.050	10.029	0.530	0.000	0.091	0.856	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	138	192	0	0	0	0	0
N.S.	1	1.00	0.37	0.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.234	10.274	0.815	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	641	297	679	0	0	0	0	0
N.S.	1	1.00	0.46	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.731	10.785	1.418	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	875	875	453	1591	0	0	0	0	0
N.S.	1	1.00	0.52	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.141	11.327	3.007	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	912	912	222	426	0	448	0	0	0
N.S.	1	1.00	0.24	0.47	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.439	10.188	8.289	0.000	0.117	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	694	694	166	323	0	332	0	0	0
N.S.	1	1.00	0.24	0.47	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.334	10.138	1.549	0.000	0.101	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	126	253	0	188	167	0	0
N.S.	1	1.00	0.32	0.64	0.00	0.48	0.42	0.00	0.00
time (sec)	N/A	0.210	10.083	0.690	0.000	0.102	4.471	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	99	212	0	115	78	0	0
N.S.	1	1.00	0.38	0.81	0.00	0.44	0.30	0.00	0.00
time (sec)	N/A	0.065	10.041	0.559	0.000	0.085	2.465	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	732	732	432	564	0	0	0	0	0
N.S.	1	1.00	0.59	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	10.660	0.865	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1494	1494	427	1384	0	0	0	0	0
N.S.	1	1.00	0.29	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.318	11.135	1.503	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2452	2452	630	2326	0	0	0	0	0
N.S.	1	1.00	0.26	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.528	11.740	3.429	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	94	80	0	59	0	0	0
N.S.	1	1.00	1.96	1.67	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.017	10.169	1.564	0.000	0.136	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	755	755	901	673	0	1041	0	0	0
N.S.	1	1.00	1.19	0.89	0.00	1.38	0.00	0.00	0.00
time (sec)	N/A	0.806	14.513	10.893	0.000	0.148	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	528	528	674	508	0	681	0	0	0
N.S.	1	1.00	1.28	0.96	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.450	12.807	4.278	0.000	0.114	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	521	409	0	435	0	0	0
N.S.	1	1.00	1.42	1.11	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.156	11.347	3.066	0.000	0.123	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	300	0	0	0
N.S.	1	1.00	1.07	1.28	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.049	0.117	0.931	0.000	0.113	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	436	436	298	359	0	0	0	0	0
N.S.	1	1.00	0.68	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.278	10.428	1.734	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	782	782	1853	1495	0	0	0	0	0
N.S.	1	1.00	2.37	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.953	12.375	3.167	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1125	1125	781	4476	0	0	0	0	0
N.S.	1	1.00	0.69	3.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.252	13.909	4.712	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	859	859	1058	1141	0	2623	0	0	0
N.S.	1	1.00	1.23	1.33	0.00	3.05	0.00	0.00	0.00
time (sec)	N/A	0.944	15.269	12.568	0.000	0.134	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	628	633	766	799	0	1669	0	0	0
N.S.	1	1.01	1.22	1.27	0.00	2.66	0.00	0.00	0.00
time (sec)	N/A	0.402	12.987	2.938	0.000	0.108	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	481	481	597	593	0	1031	0	0	0
N.S.	1	1.00	1.24	1.23	0.00	2.14	0.00	0.00	0.00
time (sec)	N/A	0.245	11.657	1.671	0.000	0.100	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	398	398	497	509	0	656	0	0	0
N.S.	1	1.00	1.25	1.28	0.00	1.65	0.00	0.00	0.00
time (sec)	N/A	0.123	10.911	1.035	0.000	0.093	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	867	1045	1736	3241	0	0	0	0	0
N.S.	1	1.21	2.00	3.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.903	13.305	1.759	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1301	2112	1116	8276	0	0	0	0	0
N.S.	1	1.62	0.86	6.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.169	15.222	3.607	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	310	365	0	0	0	0	0
N.S.	1	1.00	1.14	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	10.417	1.798	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	312	369	0	0	0	0	0
N.S.	1	1.00	1.15	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	10.408	4.428	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	74	93	0	0	0	0	0
N.S.	1	1.00	0.70	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	10.251	1.544	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	218	218	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	81	81	0	286	0	80	397
N.S.	1	1.00	0.76	0.76	0.00	2.70	0.00	0.75	3.75
time (sec)	N/A	0.087	0.024	1.168	0.000	0.331	0.000	0.309	8.632

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	139	126	0	603	0	0	0
N.S.	1	1.00	0.96	0.87	0.00	4.16	0.00	0.00	0.00
time (sec)	N/A	0.139	0.908	1.668	0.000	1.782	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	151	351	0	669	0	0	0
N.S.	1	1.00	0.63	1.47	0.00	2.80	0.00	0.00	0.00
time (sec)	N/A	0.118	0.643	2.488	0.000	1.677	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	212	245	0	469	0	358	0
N.S.	1	1.00	0.69	0.79	0.00	1.52	0.00	1.16	0.00
time (sec)	N/A	0.392	0.689	0.726	0.000	0.325	0.000	0.310	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	214	191	0	361	0	262	0
N.S.	1	1.00	0.76	0.67	0.00	1.28	0.00	0.93	0.00
time (sec)	N/A	0.210	1.190	0.581	0.000	0.301	0.000	0.333	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	174	251	0	743	0	0	0
N.S.	1	1.00	0.61	0.88	0.00	2.60	0.00	0.00	0.00
time (sec)	N/A	0.464	0.537	0.574	0.000	0.733	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	171	229	0	731	0	238	0
N.S.	1	1.00	0.58	0.78	0.00	2.49	0.00	0.81	0.00
time (sec)	N/A	0.485	0.607	0.590	0.000	0.659	0.000	0.327	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	173	214	0	749	0	360	0
N.S.	1	1.00	0.60	0.74	0.00	2.60	0.00	1.25	0.00
time (sec)	N/A	0.488	0.641	0.607	0.000	0.585	0.000	0.343	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	186	194	0	791	0	683	0
N.S.	1	1.00	0.63	0.66	0.00	2.69	0.00	2.32	0.00
time (sec)	N/A	0.486	0.877	0.614	0.000	0.595	0.000	0.351	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [14] had the largest ratio of [.392900000000000027]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	15	5	1.94	28	0.179
2	A	12	5	1.92	28	0.179
3	A	8	5	1.56	26	0.192
4	A	3	3	1.00	19	0.158
5	A	3	3	1.00	28	0.107
6	A	6	6	1.00	28	0.214
7	A	7	6	1.00	28	0.214
8	A	12	7	1.00	28	0.250
9	A	10	6	1.00	28	0.214
10	A	7	5	1.00	26	0.192
11	A	4	4	1.00	19	0.210
12	A	9	7	1.00	28	0.250
13	A	15	10	1.00	28	0.357
14	A	22	11	1.00	28	0.393
15	A	6	3	1.00	26	0.115
16	A	2	2	1.00	27	0.074
17	A	6	4	1.00	33	0.121
18	A	5	4	1.00	33	0.121
19	A	4	4	1.00	31	0.129
20	A	3	3	1.00	24	0.125
21	A	3	3	1.00	33	0.091
22	A	6	6	1.00	33	0.182
23	A	7	6	1.00	33	0.182
24	A	5	5	1.00	33	0.152

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	4	1.01	33	0.121
26	A	4	4	1.00	31	0.129
27	A	4	4	1.00	24	0.167
28	A	9	7	1.21	33	0.212
29	A	15	10	1.62	33	0.303
30	A	1	1	1.00	41	0.024
31	A	1	1	1.00	41	0.024
32	A	4	4	1.00	31	0.129
33	A	6	3	1.00	31	0.097
34	A	11	7	1.00	28	0.250
35	A	4	3	1.00	30	0.100
36	A	1	1	1.00	30	0.033
37	A	6	6	1.00	38	0.158
38	A	6	6	1.00	37	0.162
39	A	8	7	1.00	40	0.175
40	A	8	7	1.00	40	0.175
41	A	8	7	1.00	40	0.175
42	A	8	7	1.00	40	0.175

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 453

$$\begin{aligned}
&\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx \\
&= \frac{e(21Bcd^2+21Acde-5aBe^2)\,x\sqrt{a+cx^4}}{21c^2} + \frac{e^2(3Bd+Ae)x^3\sqrt{a+cx^4}}{5c} \\
&\quad + \frac{Be^3x^5\sqrt{a+cx^4}}{7c} + \frac{(5Bcd^3+15Acd^2e-9aBde^2-3aAe^3)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{\sqrt[4]{a}(5Bcd^3+15Acd^2e-9aBde^2-3aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} \\
&\quad + \frac{\left(105Ac^2d^3+25a^2Be^3-105acde(Bd+Ae)-63a^{3/2}\sqrt{c}e^2(3Bd+Ae)+105\sqrt{a}c^{3/2}d^2(Bd+3Ae)\right)\left(\sqrt{a}\right.}{210\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] 1/21*e*(21*A*c*d*e-5*B*a*e^2+21*B*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+1/5*e^2*(A*e
+3*B*d)*x^3*(c*x^4+a)^(1/2)/c+1/7*B*e^3*x^5*(c*x^4+a)^(1/2)/c+1/5*(-3*A*a*e
^3+15*A*c*d^2*e-9*B*a*d*e^2+5*B*c*d^3)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x
^2*c^(1/2))-1/5*a^(1/4)*(-3*A*a*e^3+15*A*c*d^2*e-9*B*a*d*e^2+5*B*c*d^3)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/210*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(105*A*c^2*d^3+25*a^
2*B*e^3-105*a*c*d*e*(A*e+B*d)+105*c^(3/2)*d^2*(3*A*e+B*d)*a^(1/2)-63*a^(3/2
)*e^2*(A*e+3*B*d)*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^
(1/2))^2)^(1/2)/a^(1/4)/c^(9/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.94, number of steps used = 15, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1735, 226, 311, 1210, 327}

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{Be^3\sqrt{cx^4+a}x^5}{7c}+\frac{e^2(3Bd+Ae)\sqrt{cx^4+a}x^3}{5c}-\frac{5aBe^3\sqrt{cx^4+a}x}{21c^2}$$
$$+\frac{de(Bd+Ae)\sqrt{cx^4+a}x}{c}-\frac{3ae^2(3Bd+Ae)\sqrt{cx^4+a}x}{5c^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}+\frac{d^2(Bd+3Ae)\sqrt{cx^4+a}x}{\sqrt{c}\left(\sqrt{c}x^2+\sqrt{a}\right)}$$
$$+\frac{3a^{5/4}e^2(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{cx^4+a}}$$
$$-\frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{cx^4+a}}$$
$$+\frac{Ad^3\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{cx^4+a}}$$
$$+\frac{5a^{7/4}Be^3\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{42c^{9/4}\sqrt{cx^4+a}}$$
$$-\frac{a^{3/4}de(Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{5/4}\sqrt{cx^4+a}}$$
$$-\frac{3a^{5/4}e^2(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{cx^4+a}}$$
$$+\frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{cx^4+a}}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + c*x^4],x]
```

```
[Out] (-5*a*B*e^3*x*Sqrt[a + c*x^4])/(21*c^2) + (d*e*(B*d + A*e)*x*Sqrt[a + c*x^4
])/c + (e^2*(3*B*d + A*e)*x^3*Sqrt[a + c*x^4])/(5*c) + (B*e^3*x^5*Sqrt[a +
c*x^4])/(7*c) - (3*a*e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[
a] + Sqrt[c]*x^2)) + (d^2*(B*d + 3*A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a
] + Sqrt[c]*x^2)) + (3*a^(5/4)*e^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
```

```
(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x^4]) - (a^(1/4)*d^2*(B*d + 3*A*e)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A*d^3*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) + (
5*a^(7/4)*B*e^3*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(42*c^(9/4)*Sqrt[a
+ c*x^4]) - (a^(3/4)*d*e*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*c^(5/4)*Sqrt[a + c*x^4]) - (3*a^(5/4)*e^2*(3*B*d + A*e)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4]) + (a^(1/4)*d^2*(B*d
+ 3*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4]
)
```

<u>Rule 226</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 311</u>

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

<u>Rule 327</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 1210</u>

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

<u>Rule 1735</u>

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
```

```
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{Ad^3}{\sqrt{a+cx^4}}+\frac{d^2(Bd+3Ae)x^2}{\sqrt{a+cx^4}}+\frac{3de(Bd+Ae)x^4}{\sqrt{a+cx^4}}+\frac{e^2(3Bd+Ae)x^6}{\sqrt{a+cx^4}}+\frac{Be^3x^8}{\sqrt{a+cx^4}}\right)dx\\
&= \left(Ad^3\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx+\left(Be^3\right)\int \frac{x^8}{\sqrt{a+cx^4}}\,dx+(3de(Bd+Ae))\int \frac{x^4}{\sqrt{a+cx^4}}\,dx\\
&\quad+\left(e^2(3Bd+Ae)\right)\int \frac{x^6}{\sqrt{a+cx^4}}\,dx+\left(d^2(Bd+3Ae)\right)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx\\
&= \frac{de(Bd+Ae)x\sqrt{a+cx^4}}{c}+\frac{e^2(3Bd+Ae)x^3\sqrt{a+cx^4}}{5c}+\frac{Be^3x^5\sqrt{a+cx^4}}{7c}\\
&\quad+\frac{Ad^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{\left(5aBe^3\right)\int \frac{x^4}{\sqrt{a+cx^4}}\,dx}{7c}\\
&\quad-\frac{(ade(Bd+Ae))\int \frac{1}{\sqrt{a+cx^4}}\,dx}{c}-\frac{\left(3ae^2(3Bd+Ae)\right)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{5c}\\
&\quad+\frac{\left(\sqrt{a}d^2(Bd+3Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}-\frac{\left(\sqrt{a}d^2(Bd+3Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5aBe^3x\sqrt{a+cx^4}}{21c^2} + \frac{de(Bd+Ae)x\sqrt{a+cx^4}}{c} + \frac{e^2(3Bd+Ae)x^3\sqrt{a+cx^4}}{5c} \\
&\quad + \frac{Be^3x^5\sqrt{a+cx^4}}{7c} + \frac{d^2(Bd+3Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad + \frac{Ad^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}} \\
&\quad - \frac{a^{3/4}de(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{5/4}\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} \\
&\quad + \frac{(5a^2Be^3)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{21c^2} - \frac{\left(3a^{3/2}e^2(3Bd+Ae)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}} \\
&\quad + \frac{\left(3a^{3/2}e^2(3Bd+Ae)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5aBe^3x\sqrt{a+cx^4}}{21c^2} + \frac{de(Bd+Ae)x\sqrt{a+cx^4}}{c} + \frac{e^2(3Bd+Ae)x^3\sqrt{a+cx^4}}{5c} \\
&+ \frac{Be^3x^5\sqrt{a+cx^4}}{7c} - \frac{3ae^2(3Bd+Ae)x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{d^2(Bd+3Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&+ \frac{3a^{5/4}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} \\
&- \frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&+ \frac{Ad^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}} \\
&+ \frac{5a^{7/4}Be^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{42c^{9/4}\sqrt{a+cx^4}} \\
&- \frac{a^{3/4}de(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{5/4}\sqrt{a+cx^4}} \\
&- \frac{3a^{5/4}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{a}d^2(Bd+3Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.48

$$
\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx
$$

$$
= \frac{ex(a+cx^4)\left(-25aBe^2+21Ace(5d+ex^2)+3Bc(35d^2+21dex^2+5e^2x^4)\right)+5(21Acd(cd^2-ae^2)+aBe(-
$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*(a + c*x^4)*(-25*a*B*e^2 + 21*A*c*e*(5*d + e*x^2) + 3*B*c*(35*d^2 + 21
*d*e*x^2 + 5*e^2*x^4)) + 5*(21*A*c*d*(c*d^2 - a*e^2) + a*B*e*(-21*c*d^2 + 5
*a*e^2))*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a
```

```
)] + 7*c*(5*B*c*d^3 + 15*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x^3*Sqrt[1 +
(c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(105*c^2*Sqrt[a
+ c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.69 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.72

method	result
elliptic	$\frac{B\,e^{3} x^{5} \sqrt{c\,x^{4}+a}}{7 c}+\frac{\left(e^{3} A+3 B\,e^{2} d\right) x^{3} \sqrt{c\,x^{4}+a}}{5 c}+\frac{\left(3 A d\,e^{2}+3 B\,d^{2} e-\frac{5 B\,e^{3} a}{7 c}\right) x \sqrt{c\,x^{4}+a}}{3 c}+\frac{\left(A\,d^{3}-\frac{\left(3 A d\,e^{2}+3 B\,d^{2} e-\frac{5 B\,e^{3} a}{7 c}\right) a}{3 c}\right)\ldots}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\ldots}$
risch	$\frac{e x \left(15 B\,e^{2} x^{4} c+21 A c\,e^{2} x^{2}+63 B c d e\,x^{2}+105 A c d e-25 B a\,e^{2}+105 B c\,d^{2}\right) \sqrt{c\,x^{4}+a}}{105 c^{2}}-\frac{-\frac{25 a^{2} B\,e^{3} \sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\ldots}{\ldots}$
default	$\frac{A\,d^{3} \sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+B\,e^{3} \left(\frac{x^{5} \sqrt{c\,x^{4}+a}}{7 c}-\frac{5 a x \sqrt{c\,x^{4}+a}}{21 c^{2}}+\frac{5 a^{2} \sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\ldots\right.}{21 c^{2}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\ldots\right.$

```
[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*B*e^3*x^5*(c*x^4+a)^(1/2)/c+1/5*(A*e^3+3*B*d*e^2)/c*x^3*(c*x^4+a)^(1/2)
+1/3*(3*A*d*e^2+3*B*d^2*e-5/7*B*e^3/c*a)/c*x*(c*x^4+a)^(1/2)+(A*d^3-1/3*(3*
A*d*e^2+3*B*d^2*e-5/7*B*e^3/c*a)/c*a)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(3*A*d^2*e+d^3*B-3/5*(A*e^3+3*B*d*e^2)
/c*a)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2
)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{21\,(5\,Bacd^3+15\,Aacd^2e-9\,Ba^2de^2-3\,Aa^2e^3)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-(105\,(3\,A+B)ac\ldots}{\ldots}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*(5*B*a*c*d^3 + 15*A*a*c*d^2*e - 9*B*a^2*d*e^2 - 3*A*a^2*e^3)*sqrt
(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - (105*(3*A + B)*
a*c*d^2*e - (63*A + 25*B)*a^2*e^3 + 105*(B*a*c - A*c^2)*d^3 - 21*(9*B*a^2 -
 5*A*a*c)*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x),
-1) + (15*B*a*c*e^3*x^6 + 105*B*a*c*d^3 + 315*A*a*c*d^2*e - 189*B*a^2*d*e^2
 - 63*A*a^2*e^3 + 21*(3*B*a*c*d*e^2 + A*a*c*e^3)*x^4 + 5*(21*B*a*c*d^2*e +
21*A*a*c*d*e^2 - 5*B*a^2*e^3)*x^2)*sqrt(c*x^4 + a))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{Ad^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3Ad^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3Ade^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Ae^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{Bd^3x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{3Bd^2ex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{3Bde^2x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)} + \frac{Be^3x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] A*d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + 3*A*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,),
c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*A*d*e**2*x**5*gamma(5/
4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4
)) + A*e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*p
```

```
i)/a)/(4*sqrt(a)*gamma(11/4)) + B*d**3*x**3*gamma(3/4)*hyper((1/2, 3/4), (7
/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*B*d**2*e*x**5*ga
mma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gam
ma(9/4)) + 3*B*d*e**2*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4)) + B*e**3*x**9*gamma(9/4)*hyper((1/2
, 9/4), (13/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x^2+d)^3}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(1/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(1/2), x)
```

3.2 $\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 367

$$\begin{aligned}
&\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx \\
&= \frac{e(2Bd+Ae)x\sqrt{a+cx^4}}{3c}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c}+\frac{(5Bcd^2+10Acde-3aBe^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad -\frac{\sqrt[4]{a}(5Bcd^2+10Acde-3aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} \\
&\quad +\frac{\left(15Ac^{3/2}d^2-9a^{3/2}Be^2-5a\sqrt{c}e(2Bd+Ae)+15\sqrt{a}cd(Bd+2Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{Ellipti}}{30\sqrt[4]{a}c^{7/4}\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] 1/3*e*(A*e+2*B*d)*x*(c*x^4+a)^(1/2)/c+1/5*B*e^2*x^3*(c*x^4+a)^(1/2)/c+1/5*(
10*A*c*d*e-3*B*a*e^2+5*B*c*d^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1
/2))-1/5*a^(1/4)*(10*A*c*d*e-3*B*a*e^2+5*B*c*d^2)*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x
^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/30*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(15*A*c^(3/2)*d^2-9*a^(3/2)*B*e^2+15*c*d*(2
*A*e+B*d)*a^(1/2)-5*a*e*(A*e+2*B*d)*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(7/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.92, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1735, 226, 311, 1210, 327}

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$$
$$= -\frac{a^{3/4}e(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+2Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$
$$-\frac{3a^{5/4}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{3a^{5/4}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{\sqrt[4]{a}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(2Ae+Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{a}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(2Ae+Bd)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{Ad^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$
$$+\frac{ex\sqrt{a+cx^4}(Ae+2Bd)}{3c}+\frac{dx\sqrt{a+cx^4}(2Ae+Bd)}{\sqrt{c}\,(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{3aBe^2x\sqrt{a+cx^4}}{5c^{3/2}\,(\sqrt{a}+\sqrt{c}x^2)}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*(2*B*d + A*e)*x*Sqrt[a + c*x^4])/(3*c) + (B*e^2*x^3*Sqrt[a + c*x^4])/(5*
c) - (3*a*B*e^2*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (d
*(B*d + 2*A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (3*a^
(5/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[a + c*x
^4]) - (a^(1/4)*d*(B*d + 2*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^
(3/4)*Sqrt[a + c*x^4]) + (A*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*
```

```
a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) - (3*a^(5/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*e*(2*B*d + A*e)*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4]) + (a^(1/4
)*d*(B*d + 2*A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[
c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a
 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{Ad^2}{\sqrt{a+cx^4}}+\frac{d(Bd+2Ae)x^2}{\sqrt{a+cx^4}}+\frac{e(2Bd+Ae)x^4}{\sqrt{a+cx^4}}+\frac{Be^2x^6}{\sqrt{a+cx^4}}\right)\,dx\\
&= \left(Ad^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx+\left(Be^2\right)\int \frac{x^6}{\sqrt{a+cx^4}}\,dx\\
&\quad +(e(2Bd+Ae))\int \frac{x^4}{\sqrt{a+cx^4}}\,dx+(d(Bd+2Ae))\int \frac{x^2}{\sqrt{a+cx^4}}\,dx\\
&= \frac{e(2Bd+Ae)x\sqrt{a+cx^4}}{3c}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c}\\
&\quad +\frac{Ad^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\\
&\quad -\frac{\left(3aBe^2\right)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{5c}-\frac{(ae(2Bd+Ae))\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}\\
&\quad +\frac{\left(\sqrt{a}d(Bd+2Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}-\frac{\left(\sqrt{a}d(Bd+2Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}\\
&= \frac{e(2Bd+Ae)x\sqrt{a+cx^4}}{3c}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c}+\frac{d(Bd+2Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad -\frac{\sqrt[4]{a}d(Bd+2Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}\\
&\quad +\frac{Ad^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\\
&\quad -\frac{a^{3/4}e(2Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}\\
&\quad +\frac{\sqrt[4]{a}d(Bd+2Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}\\
&\quad -\frac{\left(3a^{3/2}Be^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}}+\frac{\left(3a^{3/2}Be^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{e(2Bd+Ae)x\sqrt{a+cx^4}}{3c}+\frac{Be^2x^3\sqrt{a+cx^4}}{5c} \\
&\quad -\frac{3aBe^2x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{d(Bd+2Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad +\frac{3a^{5/4}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}} \\
&\quad -\frac{\sqrt[4]{a}d(Bd+2Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad +\frac{Ad^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}} \\
&\quad -\frac{3a^{5/4}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}} \\
&\quad -\frac{a^{3/4}e(2Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}} \\
&\quad +\frac{\sqrt[4]{a}d(Bd+2Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.43

$$
\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx
$$

$$
= \frac{ex(10Bd+5Ae+3Bex^2)(a+cx^4)-5(-3Acd^2+2aBde+aAe^2)x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2}\right.}{15c\sqrt{a+cx^4}}
$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*(10*B*d + 5*A*e + 3*B*e*x^2)*(a + c*x^4) - 5*(-3*A*c*d^2 + 2*a*B*d*e +
 a*A*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/
a)] + (5*B*c*d^2 + 10*A*c*d*e - 3*a*B*e^2)*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(15*c*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.02 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.70

method	result
elliptic	$\frac{B e^{2} x^{3} \sqrt{c x^{4}+a}}{5 c}+\frac{\left(A e^{2}+2 B e d\right) x \sqrt{c x^{4}+a}}{3 c}+\frac{\left(d^{2} A-\frac{a \left(A e^{2}+2 B e d\right)}{3 c}\right) \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^{4}+a}}+\frac{i \left(2 A e d+B d^{2}-\cdots\right.}{\cdots}$
risch	$\frac{e x \left(3 B e x^{2}+5 A e+10 B d\right) \sqrt{c x^{4}+a}}{15 c}-\frac{\frac{5 a A e^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^{4}+a}}-\frac{15 A c d^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^{4}+a}}+\frac{10 a B c \cdots}{\cdots}}{\cdots}$
default	$\frac{d^{2} A \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^{4}+a}}+B e^{2} \left(\frac{x^{3} \sqrt{c x^{4}+a}}{5 c}-\frac{3 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-E\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i \cdots\right.\right.}{5 c^{\frac{3}{2}} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^{4}+a}}\right.$

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*e^2*x^3*(c*x^4+a)^(1/2)/c+1/3*(A*e^2+2*B*d*e)/c*x*(c*x^4+a)^(1/2)+(d^
2*A-1/3*a/c*(A*e^2+2*B*d*e))/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*c^(1/2))^(1/2),I)+I*(2*A*e*d+B*d^2-3/5*a/c*B*e^2)*a^(1/2)/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Ell
ipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.56

$$\int \frac{\left(A+Bx^2\right)\left(d+ex^2\right)^2}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{3\left(5\,Bacd^2+10\,Aacde-3\,Ba^2e^2\right)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,\middle|\,-1\right)-\left(10\left(3\,A+B\right)acde+15\left(Bac-A\cdots\right.\right.}{\cdots}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*(5*B*a*c*d^2 + 10*A*a*c*d*e - 3*B*a^2*e^2)*sqrt(c)*x*(-a/c)^(3/4)*e
lliptic_e(arcsin((-a/c)^(1/4)/x), -1) - (10*(3*A + B)*a*c*d*e + 15*(B*a*c -
 A*c^2)*d^2 - (9*B*a^2 - 5*A*a*c)*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(ar
csin((-a/c)^(1/4)/x), -1) + (3*B*a*c*e^2*x^4 + 15*B*a*c*d^2 + 30*A*a*c*d*e
- 9*B*a^2*e^2 + 5*(2*B*a*c*d*e + A*a*c*e^2)*x^2)*sqrt(c*x^4 + a))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.71

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \frac{Ad^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Adex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Ae^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Bd^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)} + \frac{Be^2x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] A*d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + A*d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**
4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + A*e**2*x**5*gamma(5/4)*hyper(
(1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + B*d*
*2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*s
qrt(a)*gamma(7/4)) + B*d*e*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4
*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + B*e**2*x**7*gamma(7/4)*hyper((
1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^2}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(1/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(1/2), x)
```

3.3 $\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 277

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \frac{Bex\sqrt{a+cx^4}}{3c} + \frac{(Bd+Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}\left(3\sqrt{c}(Bd+Ae)+\frac{3Acd-aBe}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

```
[Out] 1/3*B*e*x*(c*x^4+a)^(1/2)/c+(A*e+B*d)*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^
2*c^(1/2))-a^(1/4)*(A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
c^(3/4)/(c*x^4+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*((3*A*c*d-B*a*e)/a^(1/2)+3*(A*e+B*d)*c^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(5/4)/(c*x^4+a)^(1/2
)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.56, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1735, 226, 311, 1210, 327}

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$
$$= -\frac{a^{3/4}Be(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$
$$+\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+Bd)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{Ad(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$
$$+\frac{x\sqrt{a+cx^4}(Ae+Bd)}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{Bex\sqrt{a+cx^4}}{3c}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2))/Sqrt[a + c*x^4],x]
```

```
[Out] (B*e*x*Sqrt[a + c*x^4])/(3*c) + ((B*d + A*e)*x*Sqrt[a + c*x^4])/(Sqrt[c]*(S
qrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (A*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B*e*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(B*d + A*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{Ad}{\sqrt{a+cx^4}}+\frac{(Bd+Ae)x^2}{\sqrt{a+cx^4}}+\frac{Bex^4}{\sqrt{a+cx^4}}\right) dx \\
&= (Ad)\int \frac{1}{\sqrt{a+cx^4}}\,dx+(Be)\int \frac{x^4}{\sqrt{a+cx^4}}\,dx+(Bd+Ae)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx \\
&= \frac{Bex\sqrt{a+cx^4}}{3c}+\frac{Ad\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}} \\
&\quad -\frac{(aBe)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c}+\frac{\left(\sqrt{a}(Bd+Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}-\frac{\left(\sqrt{a}(Bd+Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}
\end{aligned}$$

$$= \frac{Bex\sqrt{a+cx^4}}{3c} + \frac{(Bd+Ae)x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{Ad\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}} - \frac{a^{3/4}Be\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.44

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{Bex(a+cx^4) + (3Acd-aBe)x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right) + c(Bd+Ae)x^3\sqrt{1+\frac{cx^4}{a}}\,\text{H}}{3c\sqrt{a+cx^4}}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/Sqrt[a + c*x^4],x]
```

```
[Out] (B*e*x*(a + c*x^4) + (3*A*c*d - a*B*e)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric
2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + c*(B*d + A*e)*x^3*Sqrt[1 + (c*x^4)/a]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/(3*c*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.73

method	result
elliptic	$\frac{Bex\sqrt{c\,x^4+a}}{3c}+\frac{\left(Ad-\frac{aeB}{3c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i(Ae+Bd)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
default	$\frac{Ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+eB\left(\frac{x\sqrt{c\,x^4+a}}{3c}-\frac{a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+\frac{i(Ae+Bd)\sqrt{a}\,\sqrt{}}{}$
risch	$\frac{Bex\sqrt{c\,x^4+a}}{3c}+\frac{\frac{3Acd\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{Bae\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{i(3Ace+3Bcd)\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{}}{\sqrt{}}}{3c}$

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*e*x*(c*x^4+a)^(1/2)/c+(A*d-1/3*a/c*e*B)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2
)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(A*e+B*d)*a^(1/2)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Ellipt
icE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.46

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{3\,(Bad+Aae)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}\,E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-((3\,A+B)ae+3\,(Ba-Ac)d)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}\,F(\arcsin}{3\,acx}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(B*a*d + A*a*e)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4
)/x), -1) - ((3*A + B)*a*e + 3*(B*a - A*c)*d)*sqrt(c)*x*(-a/c)^(3/4)*ellipt
ic_f(arcsin((-a/c)^(1/4)/x), -1) + (B*a*e*x^2 + 3*B*a*d + 3*A*a*e)*sqrt(c*x
^4 + a))/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.60

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] A*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqr
t(a)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp
_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((1/2, 3/
4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + B*e*x**5*gam
ma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(9/4))
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+cx^4}}\,dx = \int \frac{\left(B\,x^2+A\right)\,\left(e\,x^2+d\right)}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(1/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(1/2), x)
```

3.4 $\int \frac{A+Bx^2}{\sqrt{a+cx^4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\begin{aligned}
&\int \frac{A+Bx^2}{\sqrt{a+cx^4}} \, dx \\
&= \frac{Bx\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{a}\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] B*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*B*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B+A
*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1212, 226, 1210}

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{Bx\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(A + B*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*(B +
 (A*Sqrt[c])/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = \left(A+\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx-\frac{\left(\sqrt{a}B\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}$$

$$=\frac{Bx\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}$$
$$+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.34

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{\sqrt{1+\frac{cx^4}{a}}\left(3Ax\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+Bx^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

```
[In] Integrate[(A + B*x^2)/Sqrt[a + c*x^4],x]
[Out] (Sqrt[1 + (c*x^4)/a]*(3*A*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ B*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4]
)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{iB\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	169
elliptic	$\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{iB\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$	169

```
[In] int((B*x^2+A)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I
*B*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.40

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \frac{Ba\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)-(Ba-Ac)\sqrt{c}x\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\sqrt{cx^4+a}Ba}{acx}$$

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (B*a*sqrt(c)*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - (B*a -
 A*c)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + sqrt(
c*x^4 + a)*B*a)/(a*c*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.35

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx = \frac{Ax\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((B*x**2+A)/(c*x**4+a)**(1/2),x)
```

```
[Out] A*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_pol
ar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}}\,dx$$

```
[In] int((A + B*x^2)/(a + c*x^4)^(1/2),x)
```

```
[Out] int((A + B*x^2)/(a + c*x^4)^(1/2), x)
```

3.5 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = -\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2+ae^2}} - \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}} + \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}de\left(cd^2-ae^2\right)\sqrt{a+cx^4}}$$

```
[Out] -1/2*(-A*e+B*d)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2
))/d^(1/2)/e^(1/2)/(a*e^2+c*d^2)^(1/2)-1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(B*a^(1/2)-A*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(-e*a^(1/2)+d*c^(1/2))/
(c*x^4+a)^(1/2)+1/4*a^(3/4)*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a
^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^
(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/c^(1/4)/d/e/(-a*e^2+c*d^2)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1723, 226, 1721}

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{a^{3/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}de\sqrt{a+cx^4}\,(cd^2-ae^2)} - \frac{(Bd-Ae)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2+cd^2}} - \frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}B-A\sqrt{c})\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\,(\sqrt{c}d-\sqrt{a}e)}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*((B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 + a*e^2]) - ((Sqrt[a]*B - A*Sqrt[c])*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqr
t[a]*e)*Sqrt[a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*e*(c*d^2 - a*e^2)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\sqrt{a}B - A\sqrt{c}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}d - \sqrt{a}e} + \frac{\left(\sqrt{a}(Bd - Ae)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}d - \sqrt{a}e}$$

$$= -\frac{(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2 + ae^2}} - \frac{\left(\sqrt{a}B - A\sqrt{c}\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d - \sqrt{a}e\right)\sqrt{a + cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right)(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}de\left(\sqrt{c}d - \sqrt{a}e\right)\sqrt{a + cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.37

$$\int \frac{A + Bx^2}{(d + ex^2)\sqrt{a + cx^4}}\,dx = -\frac{i\sqrt{1 + \frac{cx^4}{a}}\left(Bd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right), -1\right) + (-Bd + Ae)\,\text{EllipticPi}\left(-\frac{i\sqrt{a}e}{\sqrt{c}d}, i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}de\sqrt{a + cx^4}}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*(B*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]
]*x], -1] + (-(B*d) + A*e)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSi
nh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*Sqrt[
a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.52

method	result
default	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{(Ae-Bd)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$
elliptic	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)A}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(\cdots\right.}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\cdots}$

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+(A*e-B*d)/e/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2))
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)*(B*x^2 + A)/(c*e*x^6 + c*d*x^4 + a*e*x^2 + a*d), x
)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
[Out] int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.6 $\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 641

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+cx^4}}{2d(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e(Bd-Ae)x\sqrt{a+cx^4}}{2d(cd^2+ae^2)(d+ex^2)}$$
$$- \frac{(Bcd^3-3Acd^2e-aBde^2-aAe^3)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\sqrt{e}\,(cd^2+ae^2)^{3/2}}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d(cd^2+ae^2)\sqrt{a+cx^4}}$$
$$+ \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}}$$
$$+ \frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bcd^3-3Acd^2e-aBde^2-aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\,a\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}}$$

```
[Out] -1/4*(-A*a*e^3-3*A*c*d^2*e-B*a*d*e^2+B*c*d^3)*arctan(x*(a*e^2+c*d^2)^(1/2)/
d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(3/2)/(a*e^2+c*d^2)^(3/2)/e^(1/2)-1/2*e*
(-A*e+B*d)*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)+1/2*(-A*e+B*d)*x*c^(
1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(1/2)+x^2*c^(1/2))-1/2*a^(1/4)*c^(1
/4)*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d/(a*e^2+c*d^2
)/(c*x^4+a)^(1/2)+1/2*A*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
```

```
2)/a^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*(-A*a*e^3-3*A*c*d^2
*e-B*a*d*e^2+B*c*d^3)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e
*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c
^(1/4)/d^2/e/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1711, 1729, 1210, 1723, 226, 1721}

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$$
$$= -\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Bd-Ae)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}\left(ae^2+cd^2\right)}$$
$$+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(-aAe^3-aBde^2-3Acd^2e+Bcd^3\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)}$$
$$-\frac{\left(-aAe^3-aBde^2-3Acd^2e+Bcd^3\right)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\sqrt{e}\left(ae^2+cd^2\right)^{3/2}}$$
$$+\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$
$$+\frac{\sqrt{c}x\sqrt{a+cx^4}(Bd-Ae)}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}-\frac{ex\sqrt{a+cx^4}(Bd-Ae)}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqr
t[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*
x^2)) - ((B*c*d^3 - 3*A*c*d^2*e - a*B*d*e^2 - a*A*e^3)*ArcTan[(Sqrt[c*d^2 +
 a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*Sqrt[e]*(c*d^2 +
a*e^2)^(3/2)) - (a^(1/4)*c^(1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (A*c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x
^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*c*d^3 - 3*A*c*d^2*e - a*B*d*e^2 - a*A*e^
3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
```

```
pticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*
d^2 + a*e^2)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
```

```
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{\int \frac{-2Acd^2-aBde-aAe^2-2cd(Bd-Ae)x^2-ce(Bd-Ae)x^4}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)}\\
&= -\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}\\
&\quad -\frac{\int \frac{-\sqrt{a}c^{3/2}de(Bd-Ae)+ce\left(-2Acd^2-aBde-aAe^2\right)+\left(-2c^2de(Bd-Ae)+ce(Bd-Ae)\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2cde\left(cd^2+ae^2\right)}\\
&\quad -\frac{\left(\sqrt{a}\sqrt{c}(Bd-Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)}\\
&= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}\\
&\quad -\frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\left(cd^2+ae^2\right)\sqrt{a+cx^4}}\\
&\quad +\frac{\left(A\sqrt{c}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{d\left(\sqrt{c}d-\sqrt{a}e\right)}+\frac{\left(\sqrt{a}\left(Bcd^3-3Acd^2e-aBde^2-aAe^3\right)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{\left(d+ex^2\right)\sqrt{a+cx^4}}\,dx}{2d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)}
\end{aligned}$$

$$= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e(Bd-Ae)x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}$$
$$- \frac{\left(Bcd^3-3Acd^2e-aBde^2-aAe^3\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\sqrt{e}\left(cd^2+ae^2\right)^{3/2}}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\left(cd^2+ae^2\right)\sqrt{a+cx^4}}$$
$$+ \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+cx^4}}$$
$$+ \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(Bcd^3-3Acd^2e-aBde^2-aAe^3\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.46

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx$$
$$= \frac{\frac{de(-Bd+Ae)x(a+cx^4)}{(cd^2+ae^2)(d+ex^2)} - \frac{i\sqrt{1+\frac{cx^4}{a}}\left(i\sqrt{a}\sqrt{c}de(Bd-Ae)E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+\sqrt{c}d(\sqrt{c}d-i\sqrt{a}e)(Bd-Ae)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}(cd^2e+ae^3)}}{2d^2\sqrt{a+cx^4}}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] ((d*e*(-(B*d) + A*e)*x*(a + c*x^4))/((c*d^2 + a*e^2)*(d + e*x^2)) - (I*Sqrt
[1 + (c*x^4)/a]*(I*Sqrt[a]*Sqrt[c]*d*e*(B*d - A*e)*EllipticE[I*ArcSinh[Sqrt
[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(B*d -
A*e)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (-(B*c*d^3) +
3*A*c*d^2*e + a*B*d*e^2 + a*A*e^3)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d),
 I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(
c*d^2*e + a*e^3)))/(2*d^2*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.06

method	result
default	$\frac{B\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{ed\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{(Ae-Bd)\left(\frac{e^{2}x\sqrt{c\,x^{4}+a}}{2d\left(a\,e^{2}+c\,d^{2}\right)\left(e\,x^{2}+d\right)}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2\left(a\,e^{2}+c\,d^{2}\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}} \cdots\right)}{\cdots}$
elliptic	Expression too large to display

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)
)+(A*e-B*d)/e*(1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a
*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^
(1/2),I)-1/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*
(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/d/(a*e^2+c*d^
2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(
1/2),I)+1/2/d^2/(a*e^2+c*d^2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi
(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^
(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1
/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx=\int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
[Out] Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^2}\,dx$$

```
[In] int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
[Out] int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.7 $\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 875

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}-\frac{e(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}$$
$$+\frac{(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\sqrt{e}\left(cd^2+ae^2\right)^{5/2}}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}$$
$$+\frac{\sqrt[4]{c}(4Acd^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}$$
$$-\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right.}}}{32\sqrt[4]{a}\sqrt[4]{c}d^3e\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}$$

```
[Out] 1/16*(3*A*e*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)-B*(-a^2*d*e^4-10*a*c*d^3*e^2+
3*c^2*d^5))*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d
^(5/2)/(a*e^2+c*d^2)^(5/2)/e^(1/2)-1/4*e*(-A*e+B*d)*x*(c*x^4+a)^(1/2)/d/(a*
e^2+c*d^2)/(e*x^2+d)^2-1/8*e*(-3*A*a*e^3-9*A*c*d^2*e-B*a*d*e^2+5*B*c*d^3)*x
*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)+1/8*(-3*A*a*e^3-9*A*c*d^2*e-
B*a*d*e^2+5*B*c*d^3)*x*c^(1/2)*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(a^(1/2)
+x^2*c^(1/2))-1/8*a^(1/4)*c^(1/4)*(-3*A*a*e^3-9*A*c*d^2*e-B*a*d*e^2+5*B*c*d
^3)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
```

```
4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d^2/(a*e^2+c*d^2)^2/(c*x^
4+a)^(1/2)-1/32*(3*A*e*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)-B*(-a^2*d*e^4-10*a
*c*d^3*e^2+3*c^2*d^5))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-
e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2
))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/
c^(1/4)/d^3/e/(a*e^2+c*d^2)^2/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*c^
(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2
*c^(1/2))*(4*A*c*d^2+a*e*(3*A*e+B*d)+d*(-A*e+B*d)*a^(1/2)*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c
^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 875, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1711, 1729, 1210, 1723, 226, 1721}

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}(5Bcd^3-9Aced^2-aBe^2d-3aAe^3)\sqrt{cx^4+a}x}{8d^2(cd^2+ae^2)^2(\sqrt{c}x^2+\sqrt{a})} - \frac{e(5Bcd^3-9Aced^2-aBe^2d-3aAe^3)\sqrt{cx^4+a}x}{8d^2(cd^2+ae^2)^2(ex^2+d)} - \frac{e(Bd-Ae)\sqrt{cx^4+a}x}{4d(cd^2+ae^2)(ex^2+d)^2} + \frac{(3Ae(5c^2d^4+2ace^2d^2+a^2e^4)-B(3c^2d^5-10ace^2d^3-a^2e^4d))\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)}{16d^{5/2}\sqrt{e}(cd^2+ae^2)^{5/2}} - \frac{\sqrt[4]{a}\sqrt[4]{c}(5Bcd^3-9Aced^2-aBe^2d-3aAe^3)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2(cd^2+ae^2)^2\sqrt{cx^4+a}} + \frac{\sqrt[4]{c}\left(4Acd^2+\sqrt{a}\sqrt{c}(Bd-Ae)d+ae(Bd+3Ae)\right)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)\sqrt{cx^4+a}} - \frac{(\sqrt{c}d+\sqrt{a}e)(3Ae(5c^2d^4+2ace^2d^2+a^2e^4)-B(3c^2d^5-10ace^2d^3-a^2e^4d))(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}}{32\sqrt[4]{a}\sqrt[4]{c}d^3e(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^2\sqrt{cx^4+a}}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[c]*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4
])/(8*d^2*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqr
t[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d + e*x^2)^2) - (e*(5*B*c*d^3 - 9*A*c*d
^2*e - a*B*d*e^2 - 3*a*A*e^3)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2*(
```

```
d + e*x^2)) + ((3*A*e*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - B*(3*c^2*d^5
- 10*a*c*d^3*e^2 - a^2*d*e^4))*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt
[e]*Sqrt[a + c*x^4])]])/(16*d^(5/2)*Sqrt[e]*(c*d^2 + a*e^2)^(5/2)) - (a^(1/4
)*c^(1/4)*(5*B*c*d^3 - 9*A*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/
4)*(4*A*c*d^2 + Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d
^2 + a*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*A*e*(5*c^2*d^4 +
 2*a*c*d^2*e^2 + a^2*e^4) - B*(3*c^2*d^5 - 10*a*c*d^3*e^2 - a^2*d*e^4))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi
[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 +
 a*e^2)^2*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
```

```
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\text{integral} = -\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} - \frac{\int \frac{-4Acd^2-aBde-3aAe^2-4cd(Bd-Ae)x^2+ce(Bd-Ae)x^4}{\left(d+ex^2\right)^2\sqrt{a+cx^4}}\,dx}{4d\left(cd^2+ae^2\right)}$$

$$= -\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} - \frac{e\left(5Bcd^3-9Acd^2e-aBde^2-3aAe^3\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}$$
$$+ \frac{\int \frac{aBde\left(7cd^2+ae^2\right)+A\left(8c^2d^4+5acd^2e^2+3a^2e^4\right)+4cd\left(2Bcd^3-4Acd^2e-aBde^2-aAe^3\right)x^2+ce\left(5Bcd^3-9Acd^2e-aBde^2-3aAe^3\right)x^4}{\left(d+ex^2\right)\sqrt{a+cx^4}}}{8d^2\left(cd^2+ae^2\right)^2}$$

$$= -\frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2} - \frac{e\left(5Bcd^3-9Acd^2e-aBde^2-3aAe^3\right)x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}$$
$$+ \frac{\int \frac{\sqrt{a}c^{3/2}de\left(5Bcd^3-9Acd^2e-aBde^2-3aAe^3\right)+ce\left(aBde\left(7cd^2+ae^2\right)+A\left(8c^2d^4+5acd^2e^2+3a^2e^4\right)\right)+\left(-ce\left(cd-\sqrt{a}\sqrt{c}e\right)\left(5Bcd^3-9A\right.\right.}{\left(d+ex^2\right)\sqrt{a+cx^4}}}{8cd^2e\left(cd^2+ae^2\right)^2}$$
$$- \frac{\left(\sqrt{a}\sqrt{c}\left(5Bcd^3-9Acd^2e-aBde^2-3aAe^3\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{8d^2\left(cd^2+ae^2\right)^2}$$

$$= \frac{\sqrt{c}(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}$$
$$- \frac{e(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{c}(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}$$
$$+ \frac{\left(\sqrt{c}\left(4Acd^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)\right)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{4d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)}$$
$$- \frac{\left(\sqrt{a}(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{8d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2}$$

$$= \frac{\sqrt{c}(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)\left(d+ex^2\right)^2}$$
$$- \frac{e(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3)\,x\sqrt{a+cx^4}}{8d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}$$
$$+ \frac{(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}\sqrt{e}\left(cd^2+ae^2\right)^{5/2}}$$
$$- \frac{\sqrt[4]{a}\sqrt[4]{c}(5Bcd^3 - 9Acd^2e - aBde^2 - 3aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}\left(4Acd^2+\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)\sqrt{a+cx^4}}$$
$$- \frac{\left(\sqrt{c}d+\sqrt{a}e\right)(3Ae(5c^2d^4+2acd^2e^2+a^2e^4)-B(3c^2d^5-10acd^3e^2-a^2de^4))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\cdots}}{32\sqrt[4]{a}\sqrt[4]{c}d^3e\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^2\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.33 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.52

$$\int\frac{A+Bx^2}{\left(d+ex^2\right)^3\sqrt{a+cx^4}}\,dx$$
$$= \frac{-\frac{de^2x(a+cx^4)\left(2d(Bd-Ae)(cd^2+ae^2)+(5Bcd^3-9Acd^2e-aBde^2-3aAe^3)(d+ex^2)\right)}{(d+ex^2)^2} - \frac{i\sqrt{1+\frac{cx^4}{a}}\left(-i\sqrt{a}\sqrt{c}de(-5Bcd^3+9Acd^2e+aBde^2+3a\cdots\right.}{\cdots}}{\cdots}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-((d*e^2*x*(a + c*x^4)*(2*d*(B*d - A*e)*(c*d^2 + a*e^2) + (5*B*c*d^3 - 9*A
*c*d^2*e - a*B*d*e^2 - 3*a*A*e^3)*(d + e*x^2)))/(d + e*x^2)^2) - (I*Sqrt[1
+ (c*x^4)/a]*((-I)*Sqrt[a]*Sqrt[c]*d*e*(-5*B*c*d^3 + 9*A*c*d^2*e + a*B*d*e^
2 + 3*a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt
[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(A*e*(-7*c*d^2 + (2*I)*Sqrt[a]*Sqrt[c]*d*e
- 3*a*e^2) + B*d*(3*c*d^2 - (2*I)*Sqrt[a]*Sqrt[c]*d*e - a*e^2))*EllipticF[I
*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (3*A*e*(5*c^2*d^4 + 2*a*c*d^2*
e^2 + a^2*e^4) + B*(-3*c^2*d^5 + 10*a*c*d^3*e^2 + a^2*d*e^4))*EllipticPi[((
-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/S
qrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*e*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.01 (sec) , antiderivative size = 1591, normalized size of antiderivative = 1.82

method	result	size
default	Expression too large to display	1591
elliptic	Expression too large to display	1982

```
[In] int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e*(1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2
)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1
/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/
2/d^2/(a*e^2+c*d^2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1
/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1
/2)*c^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1
/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elli
pticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2
))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c)+(A*e-B*d)/e*(1/4*e^2*x*(c*x^4+a)^(1/
2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)*x*(c*x^4+a)^(1/2)/d^
2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/
2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*a*e^2-7/8*c^2*d/(a*e^2+c*d
^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
```

```
I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a
^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(
1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I
)+3/8*I*c^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*c^(1/2)*e^3/d^2/(a*e^2+c
*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(
1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/
2))^(1/2),I)+3/8/d^3/(a*e^2+c*d^2)^2*e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(
1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ell
ipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/
2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/(a*e^2+c*d^2)^2*e^2/d/(I/a^(1/
2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)
^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(
1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a*c+15/8*d/(
a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2))*c^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \int \frac{A+Bx^2}{\sqrt{a+cx^4}\,(d+ex^2)^3}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
[Out] Integral((A + B*x**2)/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+a}(ex^2+d)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx=\int \frac{B\,x^2+A}{\sqrt{c\,x^4+a}\,(e\,x^2+d)^3}\,dx$$

```
[In] int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
[Out] int((A + B*x^2)/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.8 $\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 912

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \frac{x(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3}{2ac^2\sqrt{a+cx^4}}$$

$$+\frac{Be^3x\sqrt{a+cx^4}}{3c^2}+\frac{e^2(3Bd+Ae)x\sqrt{a+cx^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\,x\sqrt{a+cx^4}}{2ac^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{\sqrt[4]{a}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}}$$

$$+\frac{(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

$$-\frac{a^{3/4}Be^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{6c^{9/4}\sqrt{a+cx^4}}$$

$$+\frac{\sqrt[4]{a}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}}$$

$$+\frac{e(3Bcd^2+3Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}}$$

$$+\frac{\left(Ac^2d^3+a^2Be^3-3acde(Bd+Ae)+a^{3/2}\sqrt{c}e^2(3Bd+Ae)-\sqrt{a}c^{3/2}d^2(Bd+3Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+}{(\sqrt{a}+}}}{4a^{5/4}c^{9/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(A*c*d*(-3*a*e^2+c*d^2)-a*B*e*(-a*e^2+3*c*d^2)+c*(-A*a*e^3+3*A*c*d^2*
e-3*B*a*d*e^2+B*c*d^3)*x^2)/a/c^2/(c*x^4+a)^(1/2)+1/3*B*e^3*x*(c*x^4+a)^(1/
2)/c^2+e^2*(A*e+3*B*d)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-1/2*
(-A*a*e^3+3*A*c*d^2*e-3*B*a*d*e^2+B*c*d^3)*x*(c*x^4+a)^(1/2)/a/c^(3/2)/(a^(
1/2)+x^2*c^(1/2))-a^(1/4)*e^2*(A*e+3*B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/2*(-A*a*e^3+3*A*c*d^2*e-3*B*a*d*e^2+
B*c*d^3)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(7/4)/(c*x
^4+a)^(1/2)-1/6*a^(3/4)*B*e^3*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)
/c^(9/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*e^2*(A*e+3*B*d)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2
)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/2*e*(3*A*c*d*e-B*a*e^2+3*
B*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(9/4)/(c*x
^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(A
*c^2*d^3+a^2*B*e^3-3*a*c*d*e*(A*e+B*d)-c^(3/2)*d^2*(3*A*e+B*d)*a^(1/2)+a^(3
/2)*e^2*(A*e+3*B*d)*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*
c^(1/2))^2)^(1/2)/a^(5/4)/c^(9/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{1735, 1193, 1212, 226, 1210, 311, 327\}$

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx =$$

$$-\frac{a^{3/4}B\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^3}{6c^{9/4}\sqrt{cx^4+a}}+\frac{Bx\sqrt{cx^4+a}e^3}{3c^2}$$

$$-\frac{\sqrt[4]{a}(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{c^{7/4}\sqrt{cx^4+a}}$$

$$+\frac{\sqrt[4]{a}(3Bd+Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e^2}{2c^{7/4}\sqrt{cx^4+a}}$$

$$+\frac{(3Bd+Ae)x\sqrt{cx^4+a}e^2}{c^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

$$+\frac{(3Bcd^2+3Aced-aBe^2)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}c^{9/4}\sqrt{cx^4+a}}$$

$$+\frac{(Bcd^3+3Aced^2-3aBe^2d-aAe^3)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{cx^4+a}}$$

$$+\frac{\left(Ac^2d^3-\sqrt{a}c^{3/2}(Bd+3Ae)d^2-3ace(Bd+Ae)d+a^2Be^3+a^{3/2}\sqrt{c}e^2(3Bd+Ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2 \ldots}}}{4a^{5/4}c^{9/4}\sqrt{cx^4+a}}$$

$$-\frac{(Bcd^3+3Aced^2-3aBe^2d-aAe^3)\,x\sqrt{cx^4+a}}{2ac^{3/2}\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

$$+\frac{x\left(c(Bcd^3+3Aced^2-3aBe^2d-aAe^3)\,x^2+Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)\right)}{2ac^2\sqrt{cx^4+a}}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*d*(c*d^2 - 3*a*e^2) - a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 + 3*A*c*
d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x^2))/(2*a*c^2*Sqrt[a + c*x^4]) + (B*e^3*x*S
qrt[a + c*x^4])/(3*c^2) + (e^2*(3*B*d + A*e)*x*Sqrt[a + c*x^4])/(c^(3/2)*(S
qrt[a] + Sqrt[c]*x^2)) - ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*x
*Sqrt[a + c*x^4])/(2*a*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e^2*(3*B
*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(7/4)*Sqrt[a + c*x^4])
 + ((B*c*d^3 + 3*A*c*d^2*e - 3*a*B*d*e^2 - a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) - (a^(3/4)*B*e^3*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
```

```
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*c^(9/4)*Sqrt[a + c*x^4]) + (a^(1/4)*e
^2*(3*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[a
+ c*x^4]) + (e*(3*B*c*d^2 + 3*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4]) + ((A*c^2*d^3 + a^2*B*e^3
 - 3*a*c*d*e*(B*d + A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d + A*e) - Sqrt[a]*c^(3
/2)*d^2*(B*d + 3*A*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c
^(9/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

integral

$$= \int \left(\frac{Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\,x^2}{c^2\,(a+cx^4)^{3/2}} + \frac{e(3Bcd^2+3Acde-aBe^2)}{c^2\sqrt{a+cx^4}} + \frac{e^2(3Bd+Ae)x^2}{c\sqrt{a+cx^4}} + \frac{Be^3x^4}{c\sqrt{a+cx^4}} \right) dx$$

$$= \frac{\int \frac{Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3)x^2}{(a+cx^4)^{3/2}}\,dx}{c^2} + \frac{(Be^3)\int \frac{x^4}{\sqrt{a+cx^4}}\,dx}{c} + \frac{(e^2(3Bd+Ae))\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{c} + \frac{(e(3Bcd^2+3Acde-aBe^2))\int \frac{1}{\sqrt{a+cx^4}}\,dx}{c^2}$$

$$= \frac{x(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\,x^2)}{2ac^2\sqrt{a+cx^4}} + \frac{Be^3x\sqrt{a+cx^4}}{3c^2} + \frac{e(3Bcd^2+3Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}} - \frac{\int \frac{-Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3)x^2}{\sqrt{a+cx^4}}\,dx}{2ac^2} - \frac{(aBe^3)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{3c^2} + \frac{\left(\sqrt{a}e^2(3Bd+Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{c^{3/2}} - \frac{\left(\sqrt{a}e^2(3Bd+Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{c^{3/2}}$$

$$
\begin{aligned}
&= \frac{x\left(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\,x^2\right)}{2ac^2\sqrt{a+cx^4}} \\
&\quad + \frac{Be^3x\sqrt{a+cx^4}}{3c^2} + \frac{e^2(3Bd+Ae)x\sqrt{a+cx^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad - \frac{\sqrt[4]{a}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}} \\
&\quad - \frac{a^{3/4}Be^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}} \\
&\quad + \frac{e(3Bcd^2+3Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}} \\
&\quad + \frac{(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}c^{3/2}} \\
&\quad + \frac{\left(Ac^2d^3+a^2Be^3-3acde(Bd+Ae)+a^{3/2}\sqrt{c}e^2(3Bd+Ae)-\sqrt{a}c^{3/2}d^2(Bd+3Ae)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{2ac^2}
\end{aligned}
$$

$$= \frac{x(Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\,x^2)}{2ac^2\sqrt{a+cx^4}} + \frac{Be^3x\sqrt{a+cx^4}}{3c^2} + \frac{e^2(3Bd+Ae)x\sqrt{a+cx^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\,x\sqrt{a+cx^4}}{2ac^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}} + \frac{(Bcd^3+3Acd^2e-3aBde^2-aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}} - \frac{a^{3/4}Be^3\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}e^2(3Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}} + \frac{e(3Bcd^2+3Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{9/4}\sqrt{a+cx^4}} + \frac{\left(Ac^2d^3+a^2Be^3-3acde(Bd+Ae)+a^{3/2}\sqrt{c}e^2(3Bd+Ae)-\sqrt{a}c^{3/2}d^2(Bd+3Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{(\ }}{4a^{5/4}c^{9/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \frac{3Acx(cd^3+ae^2(-3d+2ex^2))+aBex(5ae^2+c(-9d^2+18dex^2+2e^2x^4))+}{}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*A*c*x*(c*d^3 + a*e^2*(-3*d + 2*e*x^2)) + a*B*e*x*(5*a*e^2 + c*(-9*d^2 +
18*d*e*x^2 + 2*e^2*x^4)) + (a*B*e*(9*c*d^2 - 5*a*e^2) + 3*A*c*d*(c*d^2 + 3*
a*e^2))*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)
] + 2*c*(B*c*d^3 + 3*A*c*d^2*e - 9*a*B*d*e^2 - 3*a*A*e^3)*x^3*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c^2*Sqrt[a + c*
x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.29 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.47

method	result
elliptic	$-\frac{2c\left(\frac{\left(Aa\,e^{3}-3Ac\,d^{2}e+3Bad\,e^{2}-Bc\,d^{3}\right)x^{3}}{4c^{2}a}+\frac{\left(3Aacd\,e^{2}-A\,c^{2}d^{3}-a^{2}B\,e^{3}+3Bac\,d^{2}e\right)x}{4a\,c^{3}}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{B\,e^{3}x\sqrt{c\,x^{4}+a}}{3c^{2}}+\frac{\left(\frac{e\left(3Acde-Ba\,e^{2}+3Bc\,d^{2}\right)}{c^{2}}-\right.}{}$
default	$A\,d^{3}\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+B\,e^{3}\left(\frac{xa}{2c^{2}\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{x\sqrt{c\,x^{4}+a}}{3c^{2}}-\frac{5a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}}{6c^{2}}\right.$
risch	$\frac{B\,e^{3}x\sqrt{c\,x^{4}+a}}{3c^{2}}+\frac{3A\,c^{2}d^{3}\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)-a^{2}B\,e^{3}\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}}}\right.}{}$

```
[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/4/c^2*(A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2-B*c*d^3)/a*x^3+1/4/a/c^3*(3*
A*a*c*d*e^2-A*c^2*d^3-B*a^2*e^3+3*B*a*c*d^2*e)*x)/((x^4+a/c)*c)^(1/2)+1/3*B
*e^3*x*(c*x^4+a)^(1/2)/c^2+(e*(3*A*c*d*e-B*a*e^2+3*B*c*d^2)/c^2-1/2/c^2/a*(
3*A*a*c*d*e^2-A*c^2*d^3-B*a^2*e^3+3*B*a*c*d^2*e)-1/3*B*e^3/c^2*a)/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*(1/c*e^2*(
A*e+3*B*d)+1/2/c*(A*a*e^3-3*A*c*d^2*e+3*B*a*d*e^2-B*c*d^3)/a)*a^(1/2)/(I/a^
(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.49

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx =$$

$$-\frac{3\left(\left(Bac^2d^3+3\,Aac^2d^2e-9\,Ba^2cde^2-3\,Aa^2ce^3\right)x^5+\left(Ba^2cd^3+3\,Aa^2cd^2e-9\,Ba^3de^2-3\,Aa^3e^3\right)x\right)\sqrt{c}\left(-\right.}{}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(3*((B*a*c^2*d^3 + 3*A*a*c^2*d^2*e - 9*B*a^2*c*d*e^2 - 3*A*a^2*c*e^3)*
x^5 + (B*a^2*c*d^3 + 3*A*a^2*c*d^2*e - 9*B*a^3*d*e^2 - 3*A*a^3*e^3)*x)*sqrt
(c)*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1) - ((9*(A + B)*a*c^2
*d^2*e - (9*A + 5*B)*a^2*c*e^3 + 3*(B*a*c^2 + A*c^3)*d^3 - 9*(3*B*a^2*c - A
*a*c^2)*d*e^2)*x^5 + (9*(A + B)*a^2*c*d^2*e - (9*A + 5*B)*a^3*e^3 + 3*(B*a^
2*c + A*a*c^2)*d^3 - 9*(3*B*a^3 - A*a^2*c)*d*e^2)*x)*sqrt(c)*(-a/c)^(3/4)*e
lliptic_f(arcsin((-a/c)^(1/4)/x), -1) - (2*B*a^2*c*e^3*x^6 - 3*B*a^2*c*d^3
- 9*A*a^2*c*d^2*e + 27*B*a^3*d*e^2 + 9*A*a^3*e^3 + 6*(3*B*a^2*c*d*e^2 + A*a
^2*c*e^3)*x^4 + (3*A*a*c^2*d^3 - 9*B*a^2*c*d^2*e - 9*A*a^2*c*d*e^2 + 5*B*a^
3*e^3)*x^2)*sqrt(c*x^4 + a))/(a^2*c^3*x^5 + a^3*c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+a)**(3/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)**3/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^3}{\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{\left(B\,x^2+A\right)\,{\left(e\,x^2+d\right)}^3}{{\left(c\,x^4+a\right)}^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^3)/(a + c*x^4)^(3/2), x)
```

3.9 $\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 694

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x(Acd^2-2aBde-aAe^2+(Bcd^2+2Acde-aBe^2)x^2)}{2ac\sqrt{a+cx^4}}$$
$$+\frac{Be^2x\sqrt{a+cx^4}}{c^{3/2}(\sqrt{a}+\sqrt{c}x^2)}-\frac{(Bcd^2+2Acde-aBe^2)x\sqrt{a+cx^4}}{2ac^{3/2}(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{\sqrt[4]{a}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{(Bcd^2+2Acde-aBe^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{\sqrt[4]{a}Be^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}}$$
$$+\frac{e(2Bd+Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$
$$-\frac{\left(Bcd^2+2Acde-aBe^2-\frac{\sqrt{c}(Acd^2-2aBde-aAe^2)}{\sqrt{a}}\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(A*c*d^2-2*a*B*d*e-a*A*e^2+(2*A*c*d*e-B*a*e^2+B*c*d^2)*x^2)/a/c/(c*x^
4+a)^(1/2)+B*e^2*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-1/2*(2*A*c
*d*e-B*a*e^2+B*c*d^2)*x*(c*x^4+a)^(1/2)/a/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-a^(
1/4)*B*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*
```

```
x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^
(1/2)+1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(3/4)/c^(7/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*B*e^2*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/2*e*(A*e+2*B*d)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)-
1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*(B*c*d^2+2*A*c*d*e-B*a*e^2-(-A*a*e^2+A*c*d^2-2*B*a*d*e)*c^(1/2)/a^(
1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(7/4)/(c*x^4+a)^(
1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

$= \{1735, 1193, 1212, 226, 1210, 311\}$

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx =$$

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\frac{\sqrt{c}(-aAe^2-2aBde+Acd^2)}{\sqrt{a}}-aBe^2+2Acde+B\right.}{4a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

$$+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(-aBe^2+2Acde+Bcd^2)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

$$+\frac{e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+2Bd)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$

$$-\frac{x\sqrt{a+cx^4}(-aBe^2+2Acde+Bcd^2)}{2ac^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+\frac{x(x^2(-aBe^2+2Acde+Bcd^2)-aAe^2-2aBde+Acd^2)}{2ac\sqrt{a+cx^4}}$$

$$+\frac{\sqrt[4]{a}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}}$$

$$-\frac{\sqrt[4]{a}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}}+\frac{Be^2x\sqrt{a+cx^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*d^2 - 2*a*B*d*e - a*A*e^2 + (B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x^2))/(
2*a*c*Sqrt[a + c*x^4]) + (B*e^2*x*Sqrt[a + c*x^4])/(c^(3/2)*(Sqrt[a] + Sqrt
[c]*x^2)) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*c^(3/2
)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(c^(7/4)*Sqrt[a + c*x^4]) + ((B*c*d^2 + 2*A*c*d*e - a*B*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4]) + (a
^(1/4)*B*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[a + c*
x^4]) + (e*(2*B*d + A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)
*c^(5/4)*Sqrt[a + c*x^4]) - ((B*c*d^2 + 2*A*c*d*e - a*B*e^2 - (Sqrt[c]*(A*c
*d^2 - 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*a^(3/4)*c^(7/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{Acd^2 - 2aBde - aAe^2 + (Bcd^2 + 2Acde - aBe^2)\, x^2}{c\,(a + cx^4)^{3/2}} + \frac{e(2Bd + Ae)}{c\sqrt{a + cx^4}} + \frac{Be^2x^2}{c\sqrt{a + cx^4}} \right) dx$$

$$= \frac{\int \frac{Acd^2-2aBde-aAe^2+(Bcd^2+2Acde-aBe^2)x^2}{(a+cx^4)^{3/2}}\,dx}{c} + \frac{(Be^2)\int \frac{x^2}{\sqrt{a+cx^4}}\,dx}{c} + \frac{(e(2Bd+Ae))\int \frac{1}{\sqrt{a+cx^4}}\,dx}{c}$$

$$= \frac{x(Acd^2 - 2aBde - aAe^2 + (Bcd^2 + 2Acde - aBe^2)\, x^2)}{2ac\sqrt{a + cx^4}} + \frac{e(2Bd + Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a + cx^4}} - \frac{\int \frac{-Acd^2+2aBde+aAe^2+(Bcd^2+2Acde-aBe^2)x^2}{\sqrt{a+cx^4}}\,dx}{2ac} + \frac{\left(\sqrt{a}Be^2\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{c^{3/2}} - \frac{\left(\sqrt{a}Be^2\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{c^{3/2}}$$

$$= \frac{x(Acd^2 - 2aBde - aAe^2 + (Bcd^2 + 2Acde - aBe^2)\, x^2)}{2ac\sqrt{a + cx^4}} + \frac{Be^2x\sqrt{a + cx^4}}{c^{3/2}\left(\sqrt{a} + \sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}Be^2\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a + cx^4}} + \frac{\sqrt[4]{a}Be^2\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{a + cx^4}} + \frac{e(2Bd + Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a + cx^4}} + \frac{(Bcd^2 + 2Acde - aBe^2)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}c^{3/2}} - \frac{\left(Bcd^2 + 2Acde - aBe^2 - \frac{\sqrt{c}(Acd^2-2aBde-aAe^2)}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}c^{3/2}}$$

$$= \frac{x(Acd^2 - 2aBde - aAe^2 + (Bcd^2 + 2Acde - aBe^2)\,x^2)}{2ac\sqrt{a+cx^4}} + \frac{Be^2x\sqrt{a+cx^4}}{c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{(Bcd^2 + 2Acde - aBe^2)\,x\sqrt{a+cx^4}}{2ac^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{a+cx^4}} + \frac{(Bcd^2 + 2Acde - aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{7/4}\sqrt{a+cx^4}} + \frac{\sqrt[4]{a}Be^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{a+cx^4}} + \frac{e(2Bd + Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}} - \frac{\left(Bcd^2 + 2Acde - aBe^2 - \frac{\sqrt{c}(Acd^2 - 2aBde - aAe^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{3/4}c^{7/4}\sqrt{a+cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.24

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx = \frac{3A(cd^2 - ae^2)\,x + 6aBex(-d+ex^2) + 3(Acd^2 + 2aBde + aAe^2)\,x\sqrt{1+\frac{cx^4}{a}}\,\mathrm{H}}{}$$

[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x]

[Out] (3*A*(c*d^2 - a*e^2)*x + 6*a*B*e*x*(-d + e*x^2) + 3*(A*c*d^2 + 2*a*B*d*e + a*A*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a
)] + 2*(B*c*d^2 + 2*A*c*d*e - 3*a*B*e^2)*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeome
tric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c*Sqrt[a + c*x^4])

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.47

method	result
elliptic	$-\frac{2c\left(-\frac{\left(2Acde-Ba\,e^2+Bc\,d^2\right)x^3}{4a\,c^2}+\frac{\left(aA\,e^2-Ac\,d^2+2aBde\right)x}{4c^2a}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\left(\frac{e(Ae+2Bd)}{c}-\frac{aA\,e^2-Ac\,d^2+2aBde}{2ac}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\cdots}\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$
default	$d^2A\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+B\,e^2\left(-\frac{x^3}{2c\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{3i\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}\cdots}}{2c^{\frac{3}{2}}\cdots}\right.$

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/4*(2*A*c*d*e-B*a*e^2+B*c*d^2)/a/c^2*x^3+1/4*(A*a*e^2-A*c*d^2+2*B*a
*d*e)/c^2/a*x)/((x^4+a/c)*c)^(1/2)+(e*(A*e+2*B*d)/c-1/2*(A*a*e^2-A*c*d^2+2*
B*a*d*e)/a/c)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)+I*(1/c*B*e^2-1/2*(2*A*c*d*e-B*a*e^2+B*c*d^2)/a/c)*a^(1/2)/(I/a^(
1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^
2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-
EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{(a+cx^4)^{3/2}}\,dx =$$

$$-\frac{((Bac^2d^2+2\,Aac^2de-3\,Ba^2ce^2)x^5+(Ba^2cd^2+2\,Aa^2cde-3\,Ba^3e^2)x)\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}E\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-\cdots\right.}{\cdots}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(((B*a*c^2*d^2 + 2*A*a*c^2*d*e - 3*B*a^2*c*e^2)*x^5 + (B*a^2*c*d^2 + 2
*A*a^2*c*d*e - 3*B*a^3*e^2)*x)*sqrt(c)*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c
)^(1/4)/x), -1) - ((2*(A + B)*a*c^2*d*e + (B*a*c^2 + A*c^3)*d^2 - (3*B*a^2*
c - A*a*c^2)*e^2)*x^5 + (2*(A + B)*a^2*c*d*e + (B*a^2*c + A*a*c^2)*d^2 - (3
*B*a^3 - A*a^2*c)*e^2)*x)*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/
4)/x), -1) - (2*B*a^2*c*e^2*x^4 - B*a^2*c*d^2 - 2*A*a^2*c*d*e + 3*B*a^3*e^2
 + (A*a*c^2*d^2 - 2*B*a^2*c*d*e - A*a^2*c*e^2)*x^2)*sqrt(c*x^4 + a))/(a^2*c
^3*x^5 + a^3*c^2*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{(A+Bx^2)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+a)**(3/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)**2/(a + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\left(d+ex^2\right)^2}{\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{\left(B\,x^2+A\right)\,\left(e\,x^2+d\right)^2}{\left(c\,x^4+a\right)^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^2)/(a + c*x^4)^(3/2), x)
```

3.10 $\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 395

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{x(Acd-aBe+c(Bd+Ae)x^2)}{2ac\sqrt{a+cx^4}} - \frac{(Bd+Ae)x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+ \frac{(Bd+Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$
$$+ \frac{Be\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$
$$+ \frac{\left(Acd-aBe-\sqrt{a}\sqrt{c}(Bd+Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{5/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(A*c*d-B*a*e+c*(A*e+B*d)*x^2)/a/c/(c*x^4+a)^(1/2)-1/2*(A*e+B*d)*x*(c*
x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+1/2*(A*e+B*d)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*B*e*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(
(c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)+1/
4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(
1/2))*(A*c*d-B*a*e-(A*e+B*d)*a^(1/2)*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(5/4)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1735, 1193, 1212, 226, 1210}

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)(-\sqrt{a}\sqrt{c}(Ae+Bd}{4a^{5/4}c^{5/4}\sqrt{a+cx^4}}$$

$$+\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Ae+Bd)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

$$-\frac{x\sqrt{a+cx^4}(Ae+Bd)}{2a\sqrt{c}\,(\sqrt{a}+\sqrt{c}x^2)}+\frac{x(-aBe+cx^2(Ae+Bd)+Acd)}{2ac\sqrt{a+cx^4}}$$

$$+\frac{Be(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a+cx^4}}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*c*d - a*B*e + c*(B*d + A*e)*x^2))/(2*a*c*Sqrt[a + c*x^4]) - ((B*d + A
*e)*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + ((B*d + A*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]
) + (B*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(5/4)*Sqrt[a
+ c*x^4]) + ((A*c*d - a*B*e - Sqrt[a]*Sqrt[c]*(B*d + A*e))*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{Acd - aBe + c(Bd + Ae)x^2}{c\left(a + cx^4\right)^{3/2}} + \frac{Be}{c\sqrt{a + cx^4}}\right) dx$$

$$= \frac{\int \frac{Acd - aBe + c(Bd+Ae)x^2}{\left(a+cx^4\right)^{3/2}}\,dx}{c} + \frac{(Be)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{c}$$

$$= \frac{x(Acd - aBe + c(Bd + Ae)x^2)}{2ac\sqrt{a + cx^4}} + \frac{Be\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a + cx^4}} - \frac{\int \frac{-Acd + aBe + c(Bd+Ae)x^2}{\sqrt{a+cx^4}}\,dx}{2ac}$$

$$= \frac{x(Acd - aBe + c(Bd + Ae)x^2)}{2ac\sqrt{a + cx^4}} + \frac{Be\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a + cx^4}} + \frac{(Bd + Ae)\int \frac{1 - \frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}} + \frac{\left(Acd - aBe - \sqrt{a}\sqrt{c}(Bd + Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2ac}$$

$$= \frac{x(Acd - aBe + c(Bd + Ae)x^2)}{2ac\sqrt{a + cx^4}} - \frac{(Bd + Ae)x\sqrt{a + cx^4}}{2a\sqrt{c}\left(\sqrt{a} + \sqrt{c}x^2\right)}$$
$$+ \frac{(Bd + Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a + cx^4}}$$
$$+ \frac{Be\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}c^{5/4}\sqrt{a + cx^4}}$$
$$+ \frac{\left(Acd - aBe - \sqrt{a}\sqrt{c}(Bd + Ae)\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{5/4}\sqrt{a + cx^4}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.32

$$\int \frac{(A + Bx^2)(d + ex^2)}{(a + cx^4)^{3/2}}\,dx = \frac{3(Acd - aBe)x + 3(Acd + aBe)x\sqrt{1 + \frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{cx^4}{a}\right)}{6ac\sqrt{a + cx^4}}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*(A*c*d - a*B*e)*x + 3*(A*c*d + a*B*e)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometr
ic2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*(B*d + A*e)*x^3*Sqrt[1 + (c*x^4)/a
]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(6*a*c*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.64

method	result
elliptic	$-\frac{2c\left(-\frac{(Ae+Bd)x^3}{4ca}-\frac{(Acd-Bae)x}{4a\,c^2}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\left(\frac{eB}{c}+\frac{Acd-Bae}{2ac}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{i(Ae+Bd)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}}{}}\cdots}{2\sqrt{a}\sqrt{\cdots}}$
default	$Ad\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+eB\left(-\frac{x}{2c\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\cdots}\right.}{2c\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right.$

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/4*(A*e+B*d)/c/a*x^3-1/4*(A*c*d-B*a*e)/a/c^2*x)/((x^4+a/c)*c)^(1/2)
+(e*B/c+1/2*(A*c*d-B*a*e)/a/c)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*(A*e+B*d)/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/
2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a
)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^
(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.48

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{((Bc^2d+Ac^2e)x^4+Bacd+Aace)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right)-(((\ldots}{}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(((B*c^2*d + A*c^2*e)*x^4 + B*a*c*d + A*a*c*e)*sqrt(a)*(-c/a)^(3/4)*ell
iptic_e(arcsin(x*(-c/a)^(1/4)), -1) - (((A + B)*c^2*d + (B*a*c + A*c^2)*e)*
x^4 + (A + B)*a*c*d + (B*a^2 + A*a*c)*e)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(ar
csin(x*(-c/a)^(1/4)), -1) + sqrt(c*x^4 + a)*((B*c^2*d + A*c^2*e)*x^3 + (A*c
^2*d - B*a*c*e)*x))/(a*c^3*x^4 + a^2*c^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.42

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \frac{Adx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{Aex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bdx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)} + \frac{Bex^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] A*d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**
(3/2)*gamma(5/4)) + A*e*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*ex
```

```
p_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*d*x**3*gamma(3/4)*hyper((3/4,
3/2), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*gamma(7/4)) + B*e*x**5*
gamma(5/4)*hyper((5/4, 3/2), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2)*
gamma(9/4))
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)}{(c\,x^4+a)^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2))/(a + c*x^4)^(3/2), x)
```

3.11 $\int \frac{A+Bx^2}{\left(a+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 262

$$\int \frac{A+Bx^2}{\left(a+cx^4\right)^{3/2}}\,dx=\frac{x\left(A+Bx^2\right)}{2a\sqrt{a+cx^4}}-\frac{Bx\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(B*x^2+A)/a/(c*x^4+a)^(1/2)-1/2*B*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2
)+x^2*c^(1/2))+1/2*B*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/
c^(3/4)/(c*x^4+a)^(1/2)-1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(B*a^(1/2)-A*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x
^2*c^(1/2))^2)^(1/2)/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1193, 1212, 226, 1210}

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = -\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B-A\sqrt{c}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}} + \frac{B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}} - \frac{Bx\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(A + B*x^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(A + B*x^2))/(2*a*Sqrt[a + c*x^4]) - (B*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^
(3/4)*c^(3/4)*Sqrt[a + c*x^4]) - ((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}} - \frac{\int \frac{-A+Bx^2}{\sqrt{a+cx^4}}\,dx}{2a} \\
&= \frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}} + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a} + \frac{B\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}} \\
&= \frac{x(A+Bx^2)}{2a\sqrt{a+cx^4}} - \frac{Bx\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad + \frac{B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} \\
&\quad + \frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.38

$$
\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \frac{3Ax + 3Ax\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right) + 2Bx^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeom}}{6a\sqrt{a+cx^4}}
$$

```
[In] Integrate[(A + B*x^2)/(a + c*x^4)^(3/2),x]
```

```
[Out] (3*A*x + 3*A*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^
4)/a)] + 2*B*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*
x^4)/a)])/(6*a*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.81

method	result
elliptic	$-\frac{2c\left(-\frac{B\,x^{3}}{4ac}-\frac{Ax}{4ac}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{A\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{iB\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}\,\sqrt{c}}$
default	$A\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+B\left(\frac{x^{3}}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right.$

```
[In] int((B*x^2+A)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/4/a*B/c*x^3-1/4*A/a/c*x)/((x^4+a/c)*c)^(1/2)+1/2*A/a/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I/a^(1/2)*B/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/
2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.44

$$\int \frac{A+Bx^2}{\left(a+cx^4\right)^{3/2}}\,dx=\frac{(Bcx^4+Ba)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\left((A+B)cx^4+(A+B)a\right)\sqrt{a}\left(-\frac{c}{a}\right)}{2\left(ac^2x^4+a^2c\right)}$$

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((B*c*x^4 + B*a)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(1/4))
, -1) - ((A + B)*c*x^4 + (A + B)*a)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(
x*(-c/a)^(1/4)), -1) + (B*c*x^3 + A*c*x)*sqrt(c*x^4 + a))/(a*c^2*x^4 + a^2*
c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.30

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx=\frac{Ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{Bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

```
[In] integrate((B*x**2+A)/(c*x**4+a)**(3/2),x)
[Out] A*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(5/4)) + B*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), c*x**4*exp_po
lar(I*pi)/a)/(4*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(c*x^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(c*x^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(c\,x^4+a)^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/(a + c*x^4)^(3/2),x)
[Out] int((A + B*x^2)/(a + c*x^4)^(3/2), x)
```

3.12 $\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 732

$$\begin{aligned}
\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx &= \frac{x(Acd+aBe+c(Bd-Ae)x^2)}{2a(cd^2+ae^2)\sqrt{a+cx^4}} \\
&- \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+cx^4}}{2a(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2+ae^2)^{3/2}} \\
&+ \frac{\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^2+ae^2)\sqrt{a+cx^4}} \\
&- \frac{\sqrt[4]{c}e(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}} \\
&+ \frac{\left(Acd+aBe-\sqrt{a}\sqrt{c}(Bd-Ae)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}(cd^2+ae^2)\sqrt{a+cx^4}} \\
&+ \frac{a^{3/4}e\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d(c^2d^4-a^2e^4)\sqrt{a+cx^4}}
\end{aligned}$$

```
[Out] -1/2*e^(3/2)*(-A*e+B*d)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4
+a)^(1/2))/(a*e^2+c*d^2)^(3/2)/d^(1/2)+1/2*x*(A*c*d+B*a*e+c*(-A*e+B*d)*x^2)
/a/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)-1/2*(-A*e+B*d)*x*c^(1/2)*(c*x^4+a)^(1/2)/a
/(a*e^2+c*d^2)/(a^(1/2)+x^2*c^(1/2))+1/2*c^(1/4)*(-A*e+B*d)*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)-1/2*c^
```

```
(1/4)*e*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a
*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/4*a^(3/4)*e*(-A*e+B*d)
*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4))
)*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2
/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2
))^2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a^2*e^4+c^2*d^4)
/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*c^(1/2))*(A*c*d+B*a*e-(-A*e+B*d)*a^(1/2)*c^(1/2))*((c*x^4+a
)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)/(c*x^4+a)^(1
/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1735, 1193, 1212, 226, 1210, 1231, 1721}

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)\left(-\sqrt{a}\sqrt{c}(Bd-A\right.}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}\left(ae^2+cd^2\right)}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Bd-Ae)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}\left(ae^2+cd^2\right)}$$
$$+\frac{a^{3/4}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{c}d\sqrt{a+cx^4}\left(c^2d^4-a^2e^4\right)}$$
$$-\frac{\sqrt[4]{c}e\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}(Bd-Ae)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)\left(ae^2+cd^2\right)}$$
$$-\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\left(ae^2+cd^2\right)^{3/2}}$$
$$-\frac{\sqrt{c}x\sqrt{a+cx^4}(Bd-Ae)}{2a\left(\sqrt{a}+\sqrt{c}x^2\right)\left(ae^2+cd^2\right)}+\frac{x\left(aBe+cx^2(Bd-Ae)+Acd\right)}{2a\sqrt{a+cx^4}\left(ae^2+cd^2\right)}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (x*(A*c*d + a*B*e + c*(B*d - A*e)*x^2))/(2*a*(c*d^2 + a*e^2)*Sqrt[a + c*x^4
]) - (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt
```

```
[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(3/2)) + (c^(1/4)
*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^2 + a*
e^2)*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a + c*x
^4]) + ((A*c*d + a*B*e - Sqrt[a]*Sqrt[c]*(B*d - A*e))*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (a
^(3/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt
[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(
1/4)*d*(c^2*d^4 - a^2*e^4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{Acd + aBe + c(Bd - Ae)x^2}{(cd^2 + ae^2)(a + cx^4)^{3/2}} + \frac{e(-Bd + Ae)}{(cd^2 + ae^2)(d + ex^2)\sqrt{a + cx^4}}\right) dx$$

$$= \frac{\int \frac{Acd+aBe+c(Bd-Ae)x^2}{(a+cx^4)^{3/2}}\,dx}{cd^2 + ae^2} - \frac{(e(Bd - Ae))\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{cd^2 + ae^2}$$

$$= \frac{x(Acd + aBe + c(Bd - Ae)x^2)}{2a(cd^2 + ae^2)\sqrt{a + cx^4}} - \frac{\int \frac{-Acd-aBe+c(Bd-Ae)x^2}{\sqrt{a+cx^4}}\,dx}{2a(cd^2 + ae^2)}$$

$$- \frac{\left(\sqrt{c}e(Bd - Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 + ae^2)} + \frac{\left(\sqrt{a}e^2(Bd - Ae)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 + ae^2)}$$

$$= \frac{x(Acd + aBe + c(Bd - Ae)x^2)}{2a(cd^2 + ae^2)\sqrt{a + cx^4}} - \frac{e^{3/2}(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2 + ae^2)^{3/2}}$$

$$- \frac{\sqrt[4]{c}e(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 + ae^2)\sqrt{a + cx^4}}$$

$$+ \frac{\sqrt[4]{a}e\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right)(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de}; 2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 + ae^2)\sqrt{a + cx^4}}$$

$$+ \frac{\left(\sqrt{c}(Bd - Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\,(cd^2 + ae^2)} + \frac{\left(Acd + aBe - \sqrt{a}\sqrt{c}(Bd - Ae)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a(cd^2 + ae^2)}$$

$$= \frac{x(Acd + aBe + c(Bd - Ae)x^2)}{2a(cd^2 + ae^2)\sqrt{a + cx^4}} - \frac{\sqrt{c}(Bd - Ae)x\sqrt{a + cx^4}}{2a(cd^2 + ae^2)\left(\sqrt{a} + \sqrt{c}x^2\right)}$$

$$- \frac{e^{3/2}(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2 + ae^2)^{3/2}}$$

$$+ \frac{\sqrt[4]{c}(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^2 + ae^2)\sqrt{a + cx^4}}$$

$$- \frac{\sqrt[4]{c}e(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 + ae^2)\sqrt{a + cx^4}}$$

$$+ \frac{\left(Acd + aBe - \sqrt{a}\sqrt{c}(Bd - Ae)\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,(cd^2 + ae^2)\sqrt{a + cx^4}}$$

$$+ \frac{\sqrt[4]{a}e\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right)(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de}; 2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}d\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 + ae^2)\sqrt{a + cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.66 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.59

$$\int \frac{A + Bx^2}{(d + ex^2)(a + cx^4)^{3/2}}\,dx = \frac{A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cd^2x + aB\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}dex + B\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cd^2x^3 - A\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}cdex^3 - \sqrt{a}\sqrt{c}d(Bd -}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*x + a*B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e*x
+ B*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^2*x^3 - A*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e
*x^3 - Sqrt[a]*Sqrt[c]*d*(B*d - A*e)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + (Sqrt[a]*B - I*A*Sqrt[c])*d*(Sqrt[c]*
d - I*Sqrt[a]*e)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/S
qrt[a]]*x], -1] + (2*I)*a*B*d*e*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a
]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*I)*a*A*e
^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[S
qrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*(c*d^2 +
 a*e^2)*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 564, normalized size of antiderivative = 0.77

method	result
default	$\frac{B\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)}{e}+\frac{(Ae-Bd)\left(-\frac{2c\left(\frac{e\,x^3}{4a\left(a\,e^2+c\,d^2\right)}-\frac{dx}{4a\left(a\,e^2+c\,d^2\right)}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{cd\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\cdots}}{2a\left(a\,e^2+c\,d^2\right)\sqrt{\cdots}}\cdots\right.}{\cdots}$
elliptic	$-\frac{2c\left(\frac{(Ae-Bd)x^3}{4a\left(a\,e^2+c\,d^2\right)}-\frac{(Acd+Bae)x}{4a\left(a\,e^2+c\,d^2\right)c}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)Acd}{2a(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)Be\cdots}{2(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(A*e-B*d)/e*(-2*c*(1/4/a*e/(a*e^2+c*d^
2)*x^3-1/4*d/a/(a*e^2+c*d^2)*x)/((x^4+a/c)*c)^(1/2)+1/2*c*d/a/(a*e^2+c*d^2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/
2*I/a^(1/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*
c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I/a^(1/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/(a*e^
2+c*d^2)*e^2/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(a+cx^4)^{\frac{3}{2}}(d+ex^2)}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+a)**(3/2),x)
[Out] Integral((A + B*x**2)/((a + c*x**4)**(3/2)*(d + e*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{B\,x^2+A}{\left(c\,x^4+a\right)^{3/2}\,\left(e\,x^2+d\right)}\,dx$$

```
[In] int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)),x)
[Out] int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)), x)
```

3.13 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 1494

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+cx^4)^{3/2}}\,dx = \frac{cx(Acd^2+2aBde-aAe^2+(Bcd^2-2Acde-aBe^2)\,x^2)}{2a\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$+\frac{\sqrt{c}e^2(Bd-Ae)x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{c}(Bcd^2-2Acde-aBe^2)\,x\sqrt{a+cx^4}}{2a\,(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^2\,(d+ex^2)}-\frac{e^{3/2}(Bd-Ae)\,(3cd^2+ae^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\,(cd^2+ae^2)^{5/2}}$$

$$-\frac{e^{3/2}(Bcd^2-2Acde-aBe^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2+ae^2)^{5/2}}$$

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$+\frac{\sqrt[4]{c}(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$-\frac{\sqrt[4]{c}e(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{a+cx^4}}$$

$$-\frac{\sqrt[4]{c}e(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$-\frac{\sqrt[4]{c}\left(Bcd^2-2Acde-aBe^2-\frac{\sqrt{c}(Acd^2+2aBde-aAe^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{4a^{3/4}\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$+\frac{e\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)\,(3cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

$$+\frac{a^{3/4}e\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\right.\right.}{4\sqrt[4]{c}d\,(cd^2-ae^2)\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

```
[Out] -1/4*e^(3/2)*(-A*e+B*d)*(a*e^2+3*c*d^2)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2
)/e^(1/2)/(c*x^4+a)^(1/2))/d^(3/2)/(a*e^2+c*d^2)^(5/2)-1/2*e^(3/2)*(-2*A*c*
d*e-B*a*e^2+B*c*d^2)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)
^(1/2))/(a*e^2+c*d^2)^(5/2)/d^(1/2)+1/2*c*x*(A*c*d^2+2*a*B*d*e-a*A*e^2+(-2*
A*c*d*e-B*a*e^2+B*c*d^2)*x^2)/a/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)-1/2*e^3*(-A
*e+B*d)*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)^2/(e*x^2+d)+1/2*e^2*(-A*e+B*d)*x*
```

```
c^(1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)^2/(a^(1/2)+x^2*c^(1/2))-1/2*(-2*A*c
*d*e-B*a*e^2+B*c*d^2)*x*c^(1/2)*(c*x^4+a)^(1/2)/a/(a*e^2+c*d^2)^2/(a^(1/2)+
x^2*c^(1/2))-1/2*a^(1/4)*c^(1/4)*e^2*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/d/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)+1/2*c^(1/4)*(-2*A*c*d*
e-B*a*e^2+B*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(a
*e^2+c*d^2)^2/(c*x^4+a)^(1/2)-1/2*c^(1/4)*e*(-A*e+B*d)*(cos(2*arctan(c^(1/4
)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arc
tan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c
*x^4+a)^(1/2)-1/2*c^(1/4)*e*(-2*A*c*d*e-B*a*e^2+B*c*d^2)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^2+c*d^2)^2/(-e*a^(1/2)+d*c^(1/2))/
(c*x^4+a)^(1/2)+1/8*e*(-A*e+B*d)*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c
^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^
(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*
c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/(a*e^2+c*d^2)^2/(-e*a^(1/2)+d*c^(1/2)
)/(c*x^4+a)^(1/2)+1/4*a^(3/4)*e*(-2*A*c*d*e-B*a*e^2+B*c*d^2)*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(si
n(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^
(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4+a)
/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(a*e^2+c*d^2)^2/(c
*x^4+a)^(1/2)-1/4*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B*c*d^2-2*A*c*d*e-B*a*e^2-(-A*a*e^2+A*c*d^2+
2*B*a*d*e)*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/
4)/(a*e^2+c*d^2)^2/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 1494, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules

used = {1735, 1193, 1212, 226, 1210, 1238, 1729, 1723, 1721, 1231}

$$\int \frac{A+Bx^2}{(d+ex^2)^2(a+cx^4)^{3/2}}\,dx = -\frac{(Bd-Ae)x\sqrt{cx^4+a}e^3}{2d(cd^2+ae^2)^2(ex^2+d)}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2d(cd^2+ae^2)^2\sqrt{cx^4+a}}$$
$$+\frac{\sqrt{c}(Bd-Ae)x\sqrt{cx^4+a}e^2}{2d(cd^2+ae^2)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}-\frac{(Bd-Ae)(3cd^2+ae^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{4d^{3/2}(cd^2+ae^2)^{5/2}}$$
$$-\frac{(Bcd^2-2Aced-aBe^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{2\sqrt{d}(cd^2+ae^2)^{5/2}}$$
$$-\frac{\sqrt[4]{c}(Bcd^2-2Aced-aBe^2)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{cx^4+a}}$$
$$-\frac{\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)\sqrt{cx^4+a}}$$
$$+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)(3cd^2+ae^2)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{cx^4+a}}$$
$$+\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bcd^2-2Aced-aBe^2)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\mathrm{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right.\right.}{4\sqrt[4]{c}d(cd^2-ae^2)(cd^2+ae^2)^2\sqrt{cx^4+a}}$$
$$+\frac{\sqrt[4]{c}(Bcd^2-2Aced-aBe^2)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^2+ae^2)^2\sqrt{cx^4+a}}$$
$$-\frac{\sqrt[4]{c}\left(Bcd^2-2Aced-aBe^2-\frac{\sqrt{c}(Acd^2+2aBed-aAe^2)}{\sqrt{a}}\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{4a^{3/4}(cd^2+ae^2)^2\sqrt{cx^4+a}}$$
$$-\frac{\sqrt{c}(Bcd^2-2Aced-aBe^2)x\sqrt{cx^4+a}}{2a(cd^2+ae^2)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}$$
$$+\frac{cx(Acd^2+2aBed-aAe^2+(Bcd^2-2Aced-aBe^2)x^2)}{2a(cd^2+ae^2)^2\sqrt{cx^4+a}}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*(a + c*x^4)^(3/2)),x]
```

```
[Out] (c*x*(A*c*d^2 + 2*a*B*d*e - a*A*e^2 + (B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x^2))
/(2*a*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqrt[
a + c*x^4])/(2*d*(c*d^2 + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (Sqrt[c]*(B*c
```

```
*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*a*(c*d^2 + a*e^2)^2*(Sqrt
[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A*e)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e
^2)^2*(d + e*x^2)) - (e^(3/2)*(B*d - A*e)*(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*
d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e
^2)^(5/2)) - (e^(3/2)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*ArcTan[(Sqrt[c*d^2 +
a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(5
/2)) - (a^(1/4)*c^(1/4)*e^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*d*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (c^(1/4)*(B*c*d^2 - 2*A*c*d*
e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*d^2 + a*
e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt[a +
 c*x^4]) - (c^(1/4)*e*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqr
t[a + c*x^4]) - (c^(1/4)*(B*c*d^2 - 2*A*c*d*e - a*B*e^2 - (Sqrt[c]*(A*c*d^2
 + 2*a*B*d*e - a*A*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(3/4)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(
B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt
[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sq
rt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sq
rt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) + (a^(3/4)*e*((Sqrt
[c]*d)/Sqrt[a] + e)^2*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d -
 Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
4*c^(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
```

```
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
```

```
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{c(Acd^2+2aBde-aAe^2+(Bcd^2-2Acde-aBe^2)\,x^2)}{(cd^2+ae^2)^2\,(a+cx^4)^{3/2}} + \frac{e(-Bd+Ae)}{(cd^2+ae^2)\,(d+ex^2)^2\sqrt{a+cx^4}} + \frac{e(-Bcd^2+2Acde+aBe^2)}{(cd^2+ae^2)^2\,(d+ex^2)\sqrt{a+cx^4}}\right) dx \\
&= \frac{c\int \frac{Acd^2+2aBde-aAe^2+(Bcd^2-2Acde-aBe^2)x^2}{(a+cx^4)^{3/2}}\,dx}{(cd^2+ae^2)^2} - \frac{(e(Bd-Ae))\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx}{cd^2+ae^2} \\
&\quad - \frac{(e(Bcd^2-2Acde-aBe^2))\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{(cd^2+ae^2)^2} \\
&= \frac{cx(Acd^2+2aBde-aAe^2+(Bcd^2-2Acde-aBe^2)\,x^2)}{2a\,(cd^2+ae^2)^2\sqrt{a+cx^4}} \\
&\quad - \frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^2\,(d+ex^2)} - \frac{c\int \frac{-Acd^2-2aBde+aAe^2+(Bcd^2-2Acde-aBe^2)x^2}{\sqrt{a+cx^4}}\,dx}{2a\,(cd^2+ae^2)^2} \\
&\quad + \frac{(e(Bd-Ae))\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2d\,(cd^2+ae^2)^2} \\
&\quad - \frac{\left(\sqrt{c}e(Bcd^2-2Acde-aBe^2)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2} \\
&\quad + \frac{\left(\sqrt{a}e^2(Bcd^2-2Acde-aBe^2)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{cx(Acd^2+2aBde-aAe^2+(Bcd^2-2Acde-aBe^2)\,x^2)}{2a\,(cd^2+ae^2)^2\sqrt{a+cx^4}} \\
&\quad - \frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^2\,(d+ex^2)} - \frac{e^{3/2}(Bcd^2-2Acde-aBe^2)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2+ae^2)^{5/2}} \\
&\quad - \frac{\sqrt[4]{c}e(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}} \\
&\quad + \frac{e\left(\sqrt{c}d+\sqrt{a}e\right)(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}\dots}{\sqrt[4]{\dots}}\right.\right.}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}} \\
&\quad + \frac{(Bd-Ae)\int\frac{\sqrt{a}c^{3/2}de^2+ce(-2cd^2-ae^2)+(2c^2de^2-ce^2(cd-\sqrt{a}\sqrt{c}e))x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2cd\,(cd^2+ae^2)^2} \\
&\quad - \frac{\left(\sqrt{a}\sqrt{c}e^2(Bd-Ae)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2d\,(cd^2+ae^2)^2} + \frac{\left(\sqrt{c}(Bcd^2-2Acde-aBe^2)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\,(cd^2+ae^2)^2} \\
&\quad - \frac{\left(\sqrt{c}\left(Bcd^2-2Acde-aBe^2-\frac{\sqrt{c}(Acd^2+2aBde-aAe^2)}{\sqrt{a}}\right)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\,(cd^2+ae^2)^2}
\end{aligned}
$$

$$= \frac{cx(Acd^2+2aBde-aAe^2+(Bcd^2-2Acde-aBe^2)\,x^2)}{2a\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$+\frac{\sqrt{c}e^2(Bd-Ae)x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{c}(Bcd^2-2Acde-aBe^2)\,x\sqrt{a+cx^4}}{2a\,(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^2\,(d+ex^2)}-\frac{e^{3/2}(Bcd^2-2Acde-aBe^2)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2+ae^2)^{5/2}}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$+\frac{\sqrt[4]{c}(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{c}e(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{c}\left(Bcd^2-2Acde-aBe^2-\frac{\sqrt{c}(Acd^2+2aBde-aAe^2)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F(2\tan^{-1}(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}}{4a^{3/4}\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$+\frac{e\left(\sqrt{c}d+\sqrt{a}e\right)(Bcd^2-2Acde-aBe^2)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}(}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$-\frac{\left(\sqrt{c}e(Bd-Ae)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)}+\frac{\left(\sqrt{a}e^2(Bd-Ae)\,(3cd^2+ae^2)\right)\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^2}$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.14 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.29

$$\int\frac{A+Bx^2}{(d+ex^2)^2\,(a+cx^4)^{3/2}}\,dx=\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}d(ae^3(-Bd+Ae)x(a+cx^4)+cdx(d+ex^2)\,(-aAe^2+Bcd^2x^2+Acd}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*(a + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*(a*e^3*(-(B*d) + A*e)*x*(a + c*x^4) + c*d*x*(d
 + e*x^2)*(-(a*A*e^2) + B*c*d^2*x^2 + A*c*d*(d - 2*e*x^2) + a*B*e*(2*d - e*
x^2))) - (d + e*x^2)*Sqrt[1 + (c*x^4)/a]*(-(Sqrt[a]*Sqrt[c]*d*(-(B*c*d^3) +
```

```
 2*A*c*d^2*e + 2*a*B*d*e^2 - a*A*e^3)*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/
Sqrt[a]]*x], -1]) + I*(Sqrt[c]*d*(Sqrt[c]*d - I*Sqrt[a]*e)*(A*c*d^2 + I*Sqr
t[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(2*B*d - A*e))*EllipticF[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + a*e*(-5*B*c*d^3 + 7*A*c*d^2*e + a*B*d*e^2 + a*
A*e^3)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/
Sqrt[a]]*x], -1])))/(2*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^3 + a*d*e^2)^2*(d +
 e*x^2)*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 1384, normalized size of antiderivative = 0.93

method	result	size
default	Expression too large to display	1384
elliptic	Expression too large to display	1664

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e*(-2*c*(1/4/a*e/(a*e^2+c*d^2)*x^3-1/4*d/a/(a*e^2+c*d^2)*x)/((x^4+a/c)*c)
^(1/2)+1/2*c*d/a/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I/a^(1/2)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I/a^(1/2
)*c^(1/2)*e/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^
2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(
1/2)*c^(1/2))^(1/2),I)+1/(a*e^2+c*d^2)*e^2/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)
*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c
^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))+(A*e-B*d)/e*(-2*c*(1/2/a*d*c*e/(a
*e^2+c*d^2)^2*x^3+1/4/a*(a*e^2-c*d^2)/(a*e^2+c*d^2)^2*x)/((x^4+a/c)*c)^(1/2
)+1/2*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)-1/(I/a^(1/2)*c^(1/2
))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c
*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*e^2*c/(a*e^2+c*d^2)^
2+1/2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)*c^2/a/(a*e^2+c*d^2)^2*d^2+I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/
2)*d*e/(a*e^2+c*d^2)^2*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-I/a^(1/2)/(
I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a*e^2+c*d^2)^2*EllipticE(x*(I/a^
(1/2)*c^(1/2))^(1/2),I)-1/2*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2
```

```
)*e^3/d/(a*e^2+c*d^2)^2*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*a^(1
/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3/d/(a*e^2+c*d^2)^2*EllipticE(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*e^4/d^2/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/
2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(
c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d
,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a+7/2*e^2/(a*e^2+c*d
^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)
)*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(a+cx^4)^{\frac{3}{2}}\,(d+ex^2)^2}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+a)**(3/2),x)
[Out] Integral((A + B*x**2)/((a + c*x**4)**(3/2)*(d + e*x**2)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+a)^{\frac{3}{2}}\,(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{(d + ex^2)^2 (a + cx^4)^{3/2}} \, dx = \int \frac{Bx^2 + A}{(cx^4 + a)^{\frac{3}{2}} (ex^2 + d)^2} \, dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{(d + ex^2)^2 (a + cx^4)^{3/2}} \, dx = \int \frac{B\,x^2 + A}{(c\,x^4 + a)^{3/2} (e\,x^2 + d)^2} \, dx$$

```
[In] int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^2),x)
[Out] int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^2), x)
```

3.14 $\int \frac{A+Bx^2}{(d+ex^2)^3 (a+cx^4)^{3/2}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 2452

$$\int \frac{A+Bx^2}{(d+ex^2)^3(a+cx^4)^{3/2}}\,dx = \frac{cx(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)+c(Bcd^3-3Acd^2e-3aBde^2+aAe\cdots}{2a(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$+\frac{3\sqrt{c}e^2(Bd-Ae)(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^3(\sqrt{a}+\sqrt{c}x^2)}+\frac{\sqrt{c}e^2(Bcd^2-2Acde-aBe^2)x\sqrt{a+cx^4}}{2d(cd^2+ae^2)^3(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{c^{3/2}(Bcd^3-3Acd^2e-3aBde^2+aAe^3)x\sqrt{a+cx^4}}{2a(cd^2+ae^2)^3(\sqrt{a}+\sqrt{c}x^2)}-\frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{4d(cd^2+ae^2)^2(d+ex^2)^2}$$
$$-\frac{3e^3(Bd-Ae)(3cd^2+ae^2)x\sqrt{a+cx^4}}{8d^2(cd^2+ae^2)^3(d+ex^2)}-\frac{e^3(Bcd^2-2Acde-aBe^2)x\sqrt{a+cx^4}}{2d(cd^2+ae^2)^3(d+ex^2)}$$
$$-\frac{e^{3/2}(3cd^2+ae^2)(Bcd^2-2Acde-aBe^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}(cd^2+ae^2)^{7/2}}$$
$$-\frac{ce^{3/2}(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}(cd^2+ae^2)^{7/2}}$$
$$-\frac{3e^{3/2}(Bd-Ae)(5c^2d^4+2acd^2e^2+a^2e^4)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{16d^{5/2}(cd^2+ae^2)^{7/2}}$$
$$-\frac{3\sqrt[4]{a}\sqrt[4]{c}e^2(Bd-Ae)(3cd^2+ae^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2(Bcd^2-2Acde-aBe^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$+\frac{c^{5/4}(Bcd^3-3Acd^2e-3aBde^2+aAe^3)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{c}e(Bd-Ae)(4cd^2-\sqrt{a}\sqrt{c}de+3ae^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{8\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$-\frac{\sqrt[4]{c}e(Bcd^2-2Acde-aBe^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}d(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^2\sqrt{a+cx^4}}$$
$$-\frac{c^{5/4}e(Bcd^3-3Acd^2e-3aBde^2+aAe^3)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$+\frac{c^{3/4}\left(Ac^2d^3-a^2Be^3-\sqrt{a}c^{3/2}d^2(Bd-3Ae)+3acde(Bd-Ae)+a^{3/2}\sqrt{c}e^2(3Bd-Ae)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\cdots}}{4a^{5/4}(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$+\frac{e(\sqrt{c}d+\sqrt{a}e)(3cd^2+ae^2)(Bcd^2-2Acde-aBe^2)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\cdots\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2+ae^2)^3\sqrt{a+cx^4}}$$

```
[Out] -1/4*e^(3/2)*(a*e^2+3*c*d^2)*(-2*A*c*d*e-B*a*e^2+B*c*d^2)*arctan(x*(a*e^2+c
*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(3/2)/(a*e^2+c*d^2)^(7/2)-3/
16*e^(3/2)*(-A*e+B*d)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctan(x*(a*e^2+c*d
^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/d^(5/2)/(a*e^2+c*d^2)^(7/2)-1/2*
c*e^(3/2)*(A*a*e^3-3*A*c*d^2*e-3*B*a*d*e^2+B*c*d^3)*arctan(x*(a*e^2+c*d^2)^
(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))/(a*e^2+c*d^2)^(7/2)/d^(1/2)+1/2*c*x*
(A*c*d*(-3*a*e^2+c*d^2)+a*B*e*(-a*e^2+3*c*d^2)+c*(A*a*e^3-3*A*c*d^2*e-3*B*a
*d*e^2+B*c*d^3)*x^2)/a/(a*e^2+c*d^2)^3/(c*x^4+a)^(1/2)-1/4*e^3*(-A*e+B*d)*x
*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)^2/(e*x^2+d)^2-3/8*e^3*(-A*e+B*d)*(a*e^2+3*
c*d^2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^3/(e*x^2+d)-1/2*e^3*(-2*A*c*d*e-
B*a*e^2+B*c*d^2)*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)^3/(e*x^2+d)-1/2*c^(3/2)*
(A*a*e^3-3*A*c*d^2*e-3*B*a*d*e^2+B*c*d^3)*x*(c*x^4+a)^(1/2)/a/(a*e^2+c*d^2)
^3/(a^(1/2)+x^2*c^(1/2))+3/8*e^2*(-A*e+B*d)*(a*e^2+3*c*d^2)*x*c^(1/2)*(c*x^
4+a)^(1/2)/d^2/(a*e^2+c*d^2)^3/(a^(1/2)+x^2*c^(1/2))+1/2*e^2*(-2*A*c*d*e-B*
a*e^2+B*c*d^2)*x*c^(1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)^3/(a^(1/2)+x^2*c^(
1/2))-3/8*a^(1/4)*c^(1/4)*e^2*(-A*e+B*d)*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/d^2/(a*e^2+c*d^2)^3/(c*x^4+a)^(1/2)-1/2*a^(1/4)
*c^(1/4)*e^2*(-2*A*c*d*e-B*a*e^2+B*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/d/(a*e^2+c*d^2)^3/(c*x^4+a)^(1/2)+1/2*c^(5/4)*(A*a*e^3-3*A*c*d
^2*e-3*B*a*d*e^2+B*c*d^3)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2
^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(
3/4)/(a*e^2+c*d^2)^3/(c*x^4+a)^(1/2)-1/2*c^(1/4)*e*(-2*A*c*d*e-B*a*e^2+B*c*
d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1
/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*
c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(a*e^2+c*d^2)^
2/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/2*c^(5/4)*e*(A*a*e^3-3*A*c*d^2*e
-3*B*a*d*e^2+B*c*d^3)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)
/(a*e^2+c*d^2)^3/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*e*(a*e^2+3*c*d^
2)*(-2*A*c*d*e-B*a*e^2+B*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))
),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)
+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)
/a^(1/4)/c^(1/4)/d^2/(a*e^2+c*d^2)^3/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
+3/32*e*(-A*e+B*d)*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arct
an(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+
x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^3/(a*e^2+c*d^2)^3/(-e*a^(1/2)+d*c^(
```

```
1/2))/(c*x^4+a)^(1/2)+1/4*c^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(A*c^2*d^3-a^2*B*e^3+3*a*c*d*e*(-A*e+B*d)-c^(3/2)*d^2*(-3*A
*e+B*d)*a^(1/2)+a^(3/2)*e^2*(-A*e+3*B*d)*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c
*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/(a*e^2+c*d^2)^3/(c*x^4+a)^(1
/2)+1/4*a^(3/4)*c^(3/4)*e*(A*a*e^3-3*A*c*d^2*e-3*B*a*d*e^2+B*c*d^3)*(cos(2*
arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(
1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c
*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d/(-a*e^2+c*d^2)/(a*e^2+c*d^2)^3/(c*
x^4+a)^(1/2)-1/8*c^(1/4)*e*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(4*c*d^2+3*a*e^2-d*e*a^(1/2)*c^(1
/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*e^2+c*d^2)^2/
(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 2452, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules

used = {1735, 1193, 1212, 226, 1210, 1238, 1711, 1729, 1723, 1721, 1231}

$$\int \frac{A+Bx^2}{(d+ex^2)^3\,(a+cx^4)^{3/2}}\,dx = -\frac{3(Bd-Ae)\,(3cd^2+ae^2)\,x\sqrt{cx^4+a}e^3}{8d^2\,(cd^2+ae^2)^3\,(ex^2+d)}$$

$$-\frac{(Bcd^2-2Aced-aBe^2)\,x\sqrt{cx^4+a}e^3}{2d\,(cd^2+ae^2)^3\,(ex^2+d)}-\frac{(Bd-Ae)x\sqrt{cx^4+a}e^3}{4d\,(cd^2+ae^2)^2\,(ex^2+d)^2}$$

$$-\frac{3\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\,(3cd^2+ae^2)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{8d^2\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}(Bcd^2-2Aced-aBe^2)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2d\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$+\frac{3\sqrt{c}(Bd-Ae)\,(3cd^2+ae^2)\,x\sqrt{cx^4+a}e^2}{8d^2\,(cd^2+ae^2)^3\,(\sqrt{c}x^2+\sqrt{a})}+\frac{\sqrt{c}(Bcd^2-2Aced-aBe^2)\,x\sqrt{cx^4+a}e^2}{2d\,(cd^2+ae^2)^3\,(\sqrt{c}x^2+\sqrt{a})}$$

$$-\frac{(3cd^2+ae^2)\,(Bcd^2-2Aced-aBe^2)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{4d^{3/2}\,(cd^2+ae^2)^{7/2}}$$

$$-\frac{c(Bcd^3-3Aced^2-3aBe^2d+aAe^3)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{2\sqrt{d}\,(cd^2+ae^2)^{7/2}}$$

$$-\frac{3(Bd-Ae)\,(5c^2d^4+2ace^2d^2+a^2e^4)\arctan\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+a}}\right)e^{3/2}}{16d^{5/2}\,(cd^2+ae^2)^{7/2}}$$

$$-\frac{\sqrt[4]{c}(Bd-Ae)\left(4cd^2-\sqrt{a}\sqrt{c}ed+3ae^2\right)(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{8\sqrt[4]{a}d^2\,(\sqrt{c}d-\sqrt{a}e)\,(cd^2+ae^2)^2\sqrt{cx^4+a}}$$

$$-\frac{\sqrt[4]{c}(Bcd^2-2Aced-aBe^2)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}d\,(\sqrt{c}d-\sqrt{a}e)\,(cd^2+ae^2)^2\sqrt{cx^4+a}}$$

$$-\frac{c^{5/4}(Bcd^3-3Aced^2-3aBe^2d+aAe^3)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\,(\sqrt{c}d-\sqrt{a}e)\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$+\frac{(\sqrt{c}d+\sqrt{a}e)\,(3cd^2+ae^2)\,(Bcd^2-2Aced-aBe^2)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\mathrm{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\,(\sqrt{c}d-\sqrt{a}e)\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$+\frac{a^{3/4}c^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bcd^3-3Aced^2-3aBe^2d+aAe^3)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\mathrm{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)}{4\sqrt{a}\sqrt{c}de}\right.}{4d\,(cd^2-ae^2)\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$+\frac{3(\sqrt{c}d+\sqrt{a}e)\,(Bd-Ae)\,(5c^2d^4+2ace^2d^2+a^2e^4)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\mathrm{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3\,(\sqrt{c}d-\sqrt{a}e)\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$+\frac{c^{5/4}(Bcd^3-3Aced^2-3aBe^2d+aAe^3)\,(\sqrt{c}x^2+\sqrt{a})\,\sqrt{\frac{cx^4+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\,(cd^2+ae^2)^3\sqrt{cx^4+a}}$$

$$c^{3/4}\left(Ac^2d^3-\sqrt{a}c^{3/2}(Bd-3Ae)d^2+3ace(Bd-Ae)d-a^2Be^3+a^{3/2}\sqrt{c}e^2(3Bd-Ae)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^3*(a + c*x^4)^(3/2)),x]

[Out] (c*x*(A*c*d*(c*d^2 - 3*a*e^2) + a*B*e*(3*c*d^2 - a*e^2) + c*(B*c*d^3 - 3*A*
c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x^2))/(2*a*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^
4]) + (3*Sqrt[c]*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^
2*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*e^2*(B*c*d^2 - 2*A*
c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[
c]*x^2)) - (c^(3/2)*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*x*Sqrt[
a + c*x^4])/(2*a*(c*d^2 + a*e^2)^3*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A
*e)*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)^2*(d + e*x^2)^2) - (3*e^3*(B*d
- A*e)*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^3*(d + e
*x^2)) - (e^3*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*x*Sqrt[a + c*x^4])/(2*d*(c*d^
2 + a*e^2)^3*(d + e*x^2)) - (e^(3/2)*(3*c*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e
 - a*B*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4]
)])/(4*d^(3/2)*(c*d^2 + a*e^2)^(7/2)) - (c*e^(3/2)*(B*c*d^3 - 3*A*c*d^2*e -
 3*a*B*d*e^2 + a*A*e^3)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqr
t[a + c*x^4])])/(2*Sqrt[d]*(c*d^2 + a*e^2)^(7/2)) - (3*e^(3/2)*(B*d - A*e)*
(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[
d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2)*(c*d^2 + a*e^2)^(7/2)) - (3*a^(1/
4)*c^(1/4)*e^2*(B*d - A*e)*(3*c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*e^2*
(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*
d*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (c^(5/4)*(B*c*d^3 - 3*A*c*d^2*e - 3*
a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*(c*
d^2 + a*e^2)^3*Sqrt[a + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(4*c*d^2 - Sqrt[a]
*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*
d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^(1/4)*e
*(B*c*d^2 - 2*A*c*d*e - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2
*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) - (c^
(5/4)*e*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)^3*S
qrt[a + c*x^4]) + (c^(3/4)*(A*c^2*d^3 - a^2*B*e^3 - Sqrt[a]*c^(3/2)*d^2*(B*
d - 3*A*e) + 3*a*c*d*e*(B*d - A*e) + a^(3/2)*Sqrt[c]*e^2*(3*B*d - A*e))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d^2 + a*e^2)^3*Sqrt[a +
c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(B*c*d^2 - 2*A*c*d*e
 - a*B*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]
```

```
*e)*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (a^(3/4)*c^(3/4)*e*((Sqrt[c]*d)/Sq
rt[a] + e)^2*(B*c*d^3 - 3*A*c*d^2*e - 3*a*B*d*e^2 + a*A*e^3)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt
[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*d*(c*d^2 - a*e^2)*(c*d^2 + a*e^2)^3*Sqrt[a + c*x^4]) + (3*e*(Sqrt[
c]*d + Sqrt[a]*e)*(B*d - A*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2
)^3*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
```

```
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
```

```
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1735

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a
*e^2, 0] && IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

integral

$$= \int \left(\frac{c(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)+c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\,x^2)}{(cd^2+ae^2)^3\,(a+cx^4)^{3/2}} + \frac{e(-Bd+Ae)}{(cd^2+ae^2)\,(d+ex^2)^3\,\sqrt{a+cx^4}} + \frac{e(-Bcd^2+2Acde+aBe^2)}{(cd^2+ae^2)^2\,(d+ex^2)^2\,\sqrt{a+cx^4}} + \frac{ce(-Bcd^3+3Acd^2e+3aBde^2-aAe^3)}{(cd^2+ae^2)^3\,(d+ex^2)\,\sqrt{a+cx^4}}\right) dx$$

$$= \frac{c\int \frac{Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)+c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)x^2}{(a+cx^4)^{3/2}}\,dx}{(cd^2+ae^2)^3} - \frac{(e(Bd-Ae))\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx}{cd^2+ae^2} - \frac{(e(Bcd^2-2Acde-aBe^2))\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx}{(cd^2+ae^2)^2} - \frac{(ce(Bcd^3-3Acd^2e-3aBde^2+aAe^3))\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{(cd^2+ae^2)^3}$$

$$
\begin{aligned}
&= \frac{cx(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)+c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\,x^2)}{2a\left(cd^2+ae^2\right)^3\sqrt{a+cx^4}} \\
&\quad - \frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{4d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)^2} - \frac{e^3(Bcd^2-2Acde-aBe^2)\,x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)^3\left(d+ex^2\right)} \\
&\quad - \frac{c\int \frac{-Acd(cd^2-3ae^2)-aBe(3cd^2-ae^2)+c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)x^2}{\sqrt{a+cx^4}}\,dx}{2a\left(cd^2+ae^2\right)^3} \\
&\quad + \frac{(e(Bd-Ae))\int \frac{-4cd^2-3ae^2+4cdex^2-ce^2x^4}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx}{4d\left(cd^2+ae^2\right)^2} \\
&\quad + \frac{(e(Bcd^2-2Acde-aBe^2))\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2d\left(cd^2+ae^2\right)^3} \\
&\quad - \frac{\left(c^{3/2}e(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^3} \\
&\quad + \frac{\left(\sqrt{a}ce^2(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2+ae^2\right)^3}
\end{aligned}
$$

$$= \frac{cx(Acd(cd^2-3ae^2)+aBe(3cd^2-ae^2)+c(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\,x^2)}{2a\,(cd^2+ae^2)^3\,\sqrt{a+cx^4}}$$
$$- \frac{e^3(Bd-Ae)x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)^2\,(d+ex^2)^2} - \frac{3e^3(Bd-Ae)\,(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^3\,(d+ex^2)}$$
$$- \frac{e^3(Bcd^2-2Acde-aBe^2)\,x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)^3\,(d+ex^2)}$$
$$- \frac{ce^{3/2}(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,(cd^2+ae^2)^{7/2}}$$
$$- \frac{c^{5/4}e(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{a}c^{3/4}e\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de};2\tan\right.}{4d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2+ae^2)^3\sqrt{a+cx^4}}$$
$$- \frac{(e(Bd-Ae))\int\frac{8c^2d^4+5acd^2e^2+3a^2e^4-4cde(4cd^2+ae^2)x^2-3ce^2(3cd^2+ae^2)x^4}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{8d^2\,(cd^2+ae^2)^3}$$
$$+ \frac{(Bcd^2-2Acde-aBe^2)\int\frac{\sqrt{a}c^{3/2}de^2+ce(-2cd^2-ae^2)+(2c^2de^2-ce^2(cd-\sqrt{a}\sqrt{c}e))x^2}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2cd\,(cd^2+ae^2)^3}$$
$$- \frac{\left(\sqrt{a}\sqrt{c}e^2(Bcd^2-2Acde-aBe^2)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2d\,(cd^2+ae^2)^3}$$
$$+ \frac{\left(c^{3/2}(Bcd^3-3Acd^2e-3aBde^2+aAe^3)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\,(cd^2+ae^2)^3}$$
$$+ \frac{\left(c\left(Ac^2d^3-a^2Be^3-\sqrt{a}c^{3/2}d^2(Bd-3Ae)+3acde(Bd-Ae)+a^{3/2}\sqrt{c}e^2(3Bd-Ae)\right)\right)\int\frac{1}{\sqrt{a+cx^4}}\,dx}{2a\,(cd^2+ae^2)^3}$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 630, normalized size of antiderivative = 0.26

$$\int \frac{A+Bx^2}{(d+ex^2)^3\,(a+cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}dx\left(-2ade^3(Bd-Ae)\,(cd^2+ae^2)\,(a+cx^4)+ae^3(-13Bcd^3+17Acd^2\right.}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^3*(a + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*x*(-2*a*d*e^3*(B*d - A*e)*(c*d^2 + a*e^2)*(a +
 c*x^4) + a*e^3*(-13*B*c*d^3 + 17*A*c*d^2*e + a*B*d*e^2 + 3*a*A*e^3)*(d + e
*x^2)*(a + c*x^4) + 4*c*d^2*(d + e*x^2)^2*(B*(-(a^2*e^3) + c^2*d^3*x^2 + 3*
a*c*d*e*(d - e*x^2)) + A*c*(c*d^2*(d - 3*e*x^2) + a*e^2*(-3*d + e*x^2)))) -
 (d + e*x^2)^2*Sqrt[1 + (c*x^4)/a]*(Sqrt[a]*Sqrt[c]*d*(3*A*e*(-4*c^2*d^4 +
7*a*c*d^2*e^2 + a^2*e^4) + B*(4*c^2*d^5 - 25*a*c*d^3*e^2 + a^2*d*e^4))*Elli
pticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(Sqrt[c]*d
 - I*Sqrt[a]*e)*(4*A*c^2*d^4 + (4*I)*Sqrt[a]*c^(3/2)*d^3*(B*d - 2*A*e) + 19
*a*c*d^2*e*(B*d - A*e) - (2*I)*a^(3/2)*Sqrt[c]*d*e^2*(3*B*d - A*e) - a^2*e^
3*(B*d + 3*A*e))*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + a*
e*(3*A*e*(21*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + B*(-35*c^2*d^5 + 26*a*c*d
^3*e^2 + a^2*d*e^4))*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[a]]*x], -1])))/(8*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*(c*d^3 + a
*d*e^2)^3*(d + e*x^2)^2*Sqrt[a + c*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.43 (sec) , antiderivative size = 2326, normalized size of antiderivative = 0.95

method	result	size
default	Expression too large to display	2326
elliptic	Expression too large to display	2596

```
[In] int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e*(-2*c*(1/2/a*d*c*e/(a*e^2+c*d^2)^2*x^3+1/4/a*(a*e^2-c*d^2)/(a*e^2+c*d^2
)^2*x)/((x^4+a/c)*c)^(1/2)+1/2*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x
^2+d)-1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2
),I)*e^2*c/(a*e^2+c*d^2)^2+1/2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(
I/a^(1/2)*c^(1/2))^(1/2),I)*c^2/a/(a*e^2+c*d^2)^2*d^2+I/a^(1/2)/(I/a^(1/2)*
c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a*e^2+c*d^2)^2*EllipticF(x*(I/a^(1/2)*c^(1
/2))^(1/2),I)-I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*d*e/(a*e^2+c
*d^2)^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*a^(1/2)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3/d/(a*e^2+c*d^2)^2*EllipticF(x*(I/a^(1/2)*c^(
1/2))^(1/2),I)+1/2*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^3/d/(
a*e^2+c*d^2)^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*e^4/d^2/(a*e^2+
c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(
```

```
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1
/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1
/2))*a+7/2*e^2/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/
2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*
(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/
(I/a^(1/2)*c^(1/2))^(1/2))*c)+(A*e-B*d)/e*(-2*c*(-1/4*c*e*(a*e^2-3*c*d^2)/a
/(a*e^2+c*d^2)^3*x^3+1/4*c*d*(3*a*e^2-c*d^2)/a/(a*e^2+c*d^2)^3*x)/((x^4+a/c
)*c)^(1/2)+1/4*e^4/(a*e^2+c*d^2)^2/d*x*(c*x^4+a)^(1/2)/(e*x^2+d)^2+1/8*e^4*
(3*a*e^2+17*c*d^2)/d^2/(a*e^2+c*d^2)^3*x*(c*x^4+a)^(1/2)/(e*x^2+d)-27/8/(I/
a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*c^2*d/
(a*e^2+c*d^2)^3*e^2+1/2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)*c^3*d^3/a/(a*e^2+c*d^2)^3-1/8/(I/a^(1/2)*c^(1/2))^(1/2
)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)
^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*e^4*c/d/(a*e^2+c*d^2)^3*a+3
/8*I*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^5/d^2/(a*e^2+c*d^2)^3
*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))
^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x
^4+a)^(1/2)*c^(5/2)*e/(a*e^2+c*d^2)^3*d^2*EllipticE(x*(I/a^(1/2)*c^(1/2))^(
1/2),I)+21/8*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^
(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*e^3/(a*e^2+c*
d^2)^3*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/2*I/a^(1/2)/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*c^(5/2)*e/(a*e^2+c*d^2)^3*d^2*EllipticF(x*(I/a^(1/2)*c^(1
/2))^(1/2),I)-21/8*I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*c^(3/2)*e^3/(a*
e^2+c*d^2)^3*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*a^(3/2)/(I/a^(1
/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2
)^(1/2)/(c*x^4+a)^(1/2)*c^(1/2)*e^5/d^2/(a*e^2+c*d^2)^3*EllipticF(x*(I/a^(1
/2)*c^(1/2))^(1/2),I)+3/8*e^6/d^3/(a*e^2+c*d^2)^3/(I/a^(1/2)*c^(1/2))^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^
(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1
/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4*e^4/(a*e^2+c*d^2)^3/d
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^
(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a*c
+63/8*e^2*d/(a*e^2+c*d^2)^3/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/
a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/
a^(1/2)*c^(1/2))^(1/2))*c^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^3\left(a+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^3\left(a+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^3\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^3\left(a+cx^4\right)^{3/2}}\,dx=\int \frac{Bx^2+A}{\left(cx^4+a\right)^{\frac{3}{2}}\left(ex^2+d\right)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + a)^(3/2)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3(a+cx^4)^{3/2}}\,dx=\int \frac{B\,x^2+A}{(c\,x^4+a)^{3/2}\,(e\,x^2+d)^3}\,dx$$

```
[In] int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^3),x)
[Out] int((A + B*x^2)/((a + c*x^4)^(3/2)*(d + e*x^2)^3), x)
```

3.15 $\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4} dx = \frac{\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a} + \frac{\left(A+\frac{\sqrt{-a}B}{\sqrt{c}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}$$

```
[Out] 1/2*x*(e*x^2+d)^q*AppellF1(1/2,-q,1,3/2,-e*x^2/d,-x^2*c^(1/2)/(-a)^(1/2))*(
A-B*(-a)^(1/2)/c^(1/2))/a/((1+e*x^2/d)^q)+1/2*x*(e*x^2+d)^q*AppellF1(1/2,1,
-q,3/2,x^2*c^(1/2)/(-a)^(1/2),-e*x^2/d)*(A+B*(-a)^(1/2)/c^(1/2))/a/((1+e*x^
2/d)^q)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1707, 441, 440}

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4} dx = \frac{x\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a} + \frac{x\left(\frac{\sqrt{-a}B}{\sqrt{c}}+A\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x]
```

```
[Out] ((A - (Sqrt[-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, -((Sq
rt[c]*x^2)/Sqrt[-a]), -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q) + ((A + (Sqrt[
-a]*B)/Sqrt[c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (Sqrt[c]*x^2)/Sqr
t[-a], -((e*x^2)/d)])/(2*a*(1 + (e*x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1707

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a
, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{\left(\sqrt{-a}B+A\sqrt{c}\right)(d+ex^2)^q}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a}-\sqrt{c}x^2\right)}+\frac{\left(\sqrt{-a}B-A\sqrt{c}\right)(d+ex^2)^q}{2\sqrt{-a}\sqrt{c}\left(\sqrt{-a}+\sqrt{c}x^2\right)}\right)dx\\
&= -\left(\frac{1}{2}\left(\frac{A}{\sqrt{-a}}+\frac{B}{\sqrt{c}}\right)\int \frac{(d+ex^2)^q}{\sqrt{-a}-\sqrt{c}x^2}\,dx\right)+\frac{1}{2}\left(\frac{aA}{(-a)^{3/2}}+\frac{B}{\sqrt{c}}\right)\int \frac{(d+ex^2)^q}{\sqrt{-a}+\sqrt{c}x^2}\,dx\\
&= -\left(\frac{1}{2}\left(\left(\frac{A}{\sqrt{-a}}+\frac{B}{\sqrt{c}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{\sqrt{-a}-\sqrt{c}x^2}\,dx\right)\\
&\quad+\frac{1}{2}\left(\left(\frac{aA}{(-a)^{3/2}}+\frac{B}{\sqrt{c}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{\sqrt{-a}+\sqrt{c}x^2}\,dx\\
&= \frac{\left(A-\frac{\sqrt{-a}B}{\sqrt{c}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2a}\\
&\quad-\frac{\left(\frac{A}{\sqrt{-a}}+\frac{B}{\sqrt{c}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};\frac{\sqrt{c}x^2}{\sqrt{-a}},-\frac{ex^2}{d}\right)}{2\sqrt{-a}}
\end{aligned}$$

Mathematica [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx=\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x]
[Out] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4), x]
```

Maple [F]

$$\int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{c\,x^4+a}dx$$

```
[In] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
[Out] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="fricas")
[Out] integral((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^q}{a+cx^4}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+a}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+a),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+cx^4}\,dx=\int \frac{(B\,x^2+A)\,(e\,x^2+d)^q}{c\,x^4+a}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^q)/(a + c*x^4), x)
```

3.16 $\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 48

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{\sqrt{2}(2+x^2)\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

```
[Out] (x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/
2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1470, 422}

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{\sqrt{2}(x^2+2)\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(2 + x^2)/((1 + x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[2]*(2 + x^2)*EllipticE[ArcTan[x], 1/2])/(Sqrt[(2 + x^2)/(1 + x^2)]*Sq
rt[2 + 3*x^2 + x^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{1+x^2}\sqrt{2+x^2}\right)\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}}\,dx}{\sqrt{2+3x^2+x^4}}\\ &= \frac{\sqrt{2}(2+x^2)\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.96

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{2x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(2 + x^2)/((1 + x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (2*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt
[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

method	result	size
risch	$\frac{x(x^2+2)}{\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	80
default	$\frac{(x^2+2)x}{\sqrt{(x^2+1)(x^2+2)}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	81
elliptic	$\frac{(x^2+2)x}{\sqrt{(x^2+1)(x^2+2)}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-E\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$	81

```
[In] int((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1
/2)*x,2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{(-i\,x^2-i)E(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)+(i\,x^2+i)F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)+2\,\sqrt{x^4+3\,x^2+2}x}{2\,(x^2+1)}$$

```
[In] integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((-I*x^2 - I)*elliptic_e(arcsin(1/2*I*sqrt(2)*x), 2) + (I*x^2 + I)*elli
ptic_f(arcsin(1/2*I*sqrt(2)*x), 2) + 2*sqrt(x^4 + 3*x^2 + 2)*x)/(x^2 + 1)
```

Sympy [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{\sqrt{(x^2+1)\,(x^2+2)}\,(x^2+1)}\,dx$$

```
[In] integrate((x**2+2)/(x**2+1)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((x**2 + 2)/(sqrt((x**2 + 1)*(x**2 + 2))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{\sqrt{x^4+3\,x^2+2}(x^2+1)}\,dx$$

```
[In] integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 + 2)/(sqrt(x^4 + 3*x^2 + 2)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{\sqrt{x^4+3\,x^2+2}(x^2+1)}\,dx$$

```
[In] integrate((x^2+2)/(x^2+1)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 + 2)/(sqrt(x^4 + 3*x^2 + 2)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x^2}{(1+x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{x^2+2}{(x^2+1)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((x^2 + 2)/((x^2 + 1)*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int((x^2 + 2)/((x^2 + 1)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.17 $\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 755

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{e(7Ace(15cd-4be)+B(105c^2d^2+24b^2e^2-ce(84bd+25ae)))\,x\sqrt{a+bx^2+cx^4}}{105c^3}$$
$$+ \frac{e^2(21Bcd-6bBe+7Ace)x^3\sqrt{a+bx^2+cx^4}}{35c^2} + \frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$
$$+ \frac{(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8bce^2(21bd+1}{105c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$- \frac{\sqrt[4]{a}(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8bce^2(21bd}{105c^{15/4}\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{\sqrt[4]{a}\Big(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8bce^2(21bd}{}$$

```
[Out] 1/105*e*(7*A*c*e*(-4*b*e+15*c*d)+B*(105*c^2*d^2+24*b^2*e^2-c*e*(25*a*e+84*b
*d)))*x*(c*x^4+b*x^2+a)^(1/2)/c^3+1/35*e^2*(7*A*c*e-6*B*b*e+21*B*c*d)*x^3*(
c*x^4+b*x^2+a)^(1/2)/c^2+1/7*B*e^3*x^5*(c*x^4+b*x^2+a)^(1/2)/c+1/105*(7*A*c
*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+B*(105*c^3*d^3-48*b^3*e^3-21
*c^2*d*e*(9*a*e+10*b*d)+8*b*c*e^2*(13*a*e+21*b*d)))*x*(c*x^4+b*x^2+a)^(1/2)
/c^(7/2)/(a^(1/2)+x^2*c^(1/2))-1/105*a^(1/4)*(7*A*c*e*(45*c^2*d^2+8*b^2*e^2
-3*c*e*(3*a*e+10*b*d))+B*(105*c^3*d^3-48*b^3*e^3-21*c^2*d*e*(9*a*e+10*b*d)+
8*b*c*e^2*(13*a*e+21*b*d)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/
```

```
2)+x^2*c^(1/2))^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)+1/210*a^(1/4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^
(1/2)+x^2*c^(1/2))*(7*A*c*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+B*(
105*c^3*d^3-48*b^3*e^3-21*c^2*d*e*(9*a*e+10*b*d)+8*b*c*e^2*(13*a*e+21*b*d))
+(7*A*c*(4*a*b*e^3-15*a*c*d*e^2+15*c^2*d^3)-a*B*e*(105*c^2*d^2+24*b^2*e^2-c
*e*(25*a*e+84*b*d)))*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2)
)^2)^(1/2)/c^(15/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1693, 1211, 1117, 1209}

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}(7Ac(4abe^3-15acde^2+15c^2d^3)-a\ldots}{\ldots}\right.}{\ldots}$$

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(7Ace(-3ce(3ae+10bd)+8b^2e^2+\ldots}{105c^{15/4}\sqrt{a+bx^2+cx^4}\ldots}$$

$$+\frac{ex\sqrt{a+bx^2+cx^4}(B(-ce(25ae+84bd)+24b^2e^2+105c^2d^2)+7Ace(15cd-4be))}{105c^3}$$

$$+\frac{x\sqrt{a+bx^2+cx^4}(7Ace(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)+B(-21c^2de(9ae+10bd)+8bce^2(13ae+\ldots}{105c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$+\frac{e^2x^3\sqrt{a+bx^2+cx^4}(7Ace-6bBe+21Bcd)}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*(7*A*c*e*(15*c*d - 4*b*e) + B*(105*c^2*d^2 + 24*b^2*e^2 - c*e*(84*b*d +
25*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^3) + (e^2*(21*B*c*d - 6*b*B*e +
 7*A*c*e)*x^3*Sqrt[a + b*x^2 + c*x^4])/(35*c^2) + (B*e^3*x^5*Sqrt[a + b*x^2
 + c*x^4])/(7*c) + ((7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*
e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2
*(21*b*d + 13*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(7/2)*(Sqrt[a] + Sqr
t[c]*x^2)) - (a^(1/4)*(7*A*c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*
a*e)) + B*(105*c^3*d^3 - 48*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e
^2*(21*b*d + 13*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(105*c^(15/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(7*A*
c*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)) + B*(105*c^3*d^3 - 48
```

```
*b^3*e^3 - 21*c^2*d*e*(10*b*d + 9*a*e) + 8*b*c*e^2*(21*b*d + 13*a*e)) + (Sq
rt[c]*(7*A*c*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^3) - a*B*e*(105*c^2*d^2 +
 24*b^2*e^2 - c*e*(84*b*d + 25*a*e))))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(15/4)*Sqrt[a + b*x^2 +
 c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c} + \frac{\int \frac{7Acd^3+7cd^2(Bd+3Ae)x^2+e\left(21Bcd^2+21Acde-5aBe^2\right)x^4+e^2(21Bcd-6bBe+7Ace)x^6}{\sqrt{a+bx^2+cx^4}}\,dx}{7c}$$

$$= \frac{e^2(21Bcd-6bBe+7Ace)x^3\sqrt{a+bx^2+cx^4}}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$
$$+\frac{\int \frac{35Ac^2d^3+(21Ace(5cd^2-ae^2)+B(35c^2d^3-63acde^2+18abe^3))x^2+e(7Ace(15cd-4be)+B(105c^2d^2+24b^2e^2-ce(84bd+25ae)))x^4}{\sqrt{a+bx^2+cx^4}}}{35c^2}$$

$$= \frac{e(7Ace(15cd-4be)+B(105c^2d^2+24b^2e^2-ce(84bd+25ae)))\,x\sqrt{a+bx^2+cx^4}}{105c^3}$$
$$+\frac{e^2(21Bcd-6bBe+7Ace)x^3\sqrt{a+bx^2+cx^4}}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$
$$+\frac{\int \frac{7Ac(15c^2d^3-15acde^2+4abe^3)-aBe(105c^2d^2+24b^2e^2-ce(84bd+25ae))+(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-}{\sqrt{a+bx^2+cx^4}}}{105c^3}$$

$$= \frac{e(7Ace(15cd-4be)+B(105c^2d^2+24b^2e^2-ce(84bd+25ae)))\,x\sqrt{a+bx^2+cx^4}}{105c^3}$$
$$+\frac{e^2(21Bcd-6bBe+7Ace)x^3\sqrt{a+bx^2+cx^4}}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$
$$-\frac{(\sqrt{a}(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8}{105c^{7/2}}$$
$$+\frac{\left(\sqrt{a}\left(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+\right.\right.}{10}$$

$$= \frac{e(7Ace(15cd-4be)+B(105c^2d^2+24b^2e^2-ce(84bd+25ae)))\,x\sqrt{a+bx^2+cx^4}}{105c^3}$$
$$+\frac{e^2(21Bcd-6bBe+7Ace)x^3\sqrt{a+bx^2+cx^4}}{35c^2}+\frac{Be^3x^5\sqrt{a+bx^2+cx^4}}{7c}$$
$$+\frac{(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8bce^2}{105c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{a}(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8b}{105c^{15/4}\sqrt{a+bx^2}}$$
$$+\frac{\sqrt[4]{a}\left(7Ace(45c^2d^2+8b^2e^2-3ce(10bd+3ae))+B(105c^3d^3-48b^3e^3-21c^2de(10bd+9ae)+8\right.}{}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.51 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}ex(a+bx^2+cx^4)\,(7Ace(-4be+3c(5d+ex^2))+B(24b^2e^2-ce(84bd+25ae+18bex^2)+3c}{}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4)*(7*A*c*e*(-4*b
*e + 3*c*(5*d + e*x^2)) + B*(24*b^2*e^2 - c*e*(84*b*d + 25*a*e + 18*b*e*x^2
) + 3*c^2*(35*d^2 + 21*d*e*x^2 + 5*e^2*x^4))) - I*(-b + Sqrt[b^2 - 4*a*c])*
(7*A*c*e*(-45*c^2*d^2 - 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)) + B*(-105*c^3*d
^3 + 48*b^3*e^3 + 21*c^2*d*e*(10*b*d + 9*a*e) - 8*b*c*e^2*(21*b*d + 13*a*e)
))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*
b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])] + I*(-48*b^4*B*e^3 + 8*b^3*e^2*(21*B*c*d + 7*A*c*e +
 6*B*Sqrt[b^2 - 4*a*c]*e) - 2*b^2*c*e*(7*A*e*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*
e) + B*(105*c*d^2 + 84*Sqrt[b^2 - 4*a*c]*d*e - 76*a*e^2)) + b*c*(B*(105*c^2
*d^3 - 104*a*Sqrt[b^2 - 4*a*c]*e^3 + 21*c*d*e*(10*Sqrt[b^2 - 4*a*c]*d - 17*
a*e)) + 7*A*c*e*(45*c*d^2 + e*(30*Sqrt[b^2 - 4*a*c]*d - 17*a*e))) + c^2*(B*
(a*e^2*(189*Sqrt[b^2 - 4*a*c]*d - 50*a*e) - 105*c*d^2*(Sqrt[b^2 - 4*a*c]*d
- 2*a*e)) - 21*A*(10*c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 + 5*c*d*e*(3*Sqrt[
b^2 - 4*a*c]*d - 2*a*e))))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(420*c^4*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 10.89 (sec) , antiderivative size = 673, normalized size of antiderivative = 0.89

method	result
elliptic	$\frac{B\,e^{3} x^{5} \sqrt{c\,x^{4}+b\,x^{2}+a}}{7 c}+\frac{\left(e^{3} A+3 B\,e^{2} d-\frac{6 B\,e^{3} b}{7 c}\right) x^{3} \sqrt{c\,x^{4}+b\,x^{2}+a}}{5 c}+\frac{\left(3 A d\,e^{2}+3 B\,d^{2} e-\frac{5 B\,e^{3} a}{7 c}-\frac{4 \left(e^{3} A+3 B\,e^{2} d-\frac{6 B\,e^{3} b}{7 c}\right) b}{5 c}\right) x \sqrt{c}}{3 c}$
risch	Expression too large to display
default	Expression too large to display

[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

```
[Out] 1/7*B*e^3*x^5*(c*x^4+b*x^2+a)^(1/2)/c+1/5*(e^3*A+3*B*e^2*d-6/7*B*e^3/c*b)/c
*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(3*A*d*e^2+3*B*d^2*e-5/7*B*e^3/c*a-4/5*(e^3*
A+3*B*e^2*d-6/7*B*e^3/c*b)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(A*d^3-1/3*(3
*A*d*e^2+3*B*d^2*e-5/7*B*e^3/c*a-4/5*(e^3*A+3*B*e^2*d-6/7*B*e^3/c*b)/c*b)/c
*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(3*A*d^2*e+d^3*B-3/5*(e^3*A+3*B*e^2*d-6/
7*B*e^3/c*b)/c*a-2/3*(3*A*d*e^2+3*B*d^2*e-5/7*B*e^3/c*a-4/5*(e^3*A+3*B*e^2*
d-6/7*B*e^3/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2
*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 1041, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/210*(sqrt(1/2)*((105*B*a*c^4*d^3 - 105*(2*B*a*b*c^3 - 3*A*a*c^4)*d^2*e +
21*(8*B*a*b^2*c^2 - (9*B*a^2 + 10*A*a*b)*c^3)*d*e^2 - (48*B*a*b^3*c + 63*A*
a^2*c^3 - 8*(13*B*a^2*b + 7*A*a*b^2)*c^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) -
(105*B*a*b*c^3*d^3 - 105*(2*B*a*b^2*c^2 - 3*A*a*b*c^3)*d^2*e + 21*(8*B*a*b^
3*c - (9*B*a^2*b + 10*A*a*b^2)*c^2)*d*e^2 - (48*B*a*b^4 + 63*A*a^2*b*c^2 -
```

```
8*(13*B*a^2*b^2 + 7*A*a*b^3)*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/
c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) -
b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2
)*((105*(B*a*c^4 - A*c^5)*d^3 - 105*(2*B*a*b*c^3 - (3*A + B)*a*c^4)*d^2*e +
 21*(8*B*a*b^2*c^2 + 5*A*a*c^4 - (9*B*a^2 + 2*(5*A + 2*B)*a*b)*c^3)*d*e^2 -
 (48*B*a*b^3*c + ((63*A + 25*B)*a^2 + 28*A*a*b)*c^3 - 8*(13*B*a^2*b + (7*A
+ 3*B)*a*b^2)*c^2)*e^3)*x*sqrt((b^2 - 4*a*c)/c^2) - (105*(B*a*b*c^3 + A*b*c
^4)*d^3 - 105*(2*B*a*b^2*c^2 - (3*A - B)*a*b*c^3)*d^2*e + 21*(8*B*a*b^3*c -
 5*A*a*b*c^3 - (9*B*a^2*b + 2*(5*A - 2*B)*a*b^2)*c^2)*d*e^2 - (48*B*a*b^4 +
 ((63*A - 25*B)*a^2*b - 28*A*a*b^2)*c^2 - 8*(13*B*a^2*b^2 + (7*A - 3*B)*a*b
^3)*c)*e^3)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(a
rcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((
b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(15*B*a*c^4*e^3*x^6 + 105*B*a*c
^4*d^3 + 3*(21*B*a*c^4*d*e^2 - (6*B*a*b*c^3 - 7*A*a*c^4)*e^3)*x^4 - 105*(2*
B*a*b*c^3 - 3*A*a*c^4)*d^2*e + 21*(8*B*a*b^2*c^2 - (9*B*a^2 + 10*A*a*b)*c^3
)*d*e^2 - (48*B*a*b^3*c + 63*A*a^2*c^3 - 8*(13*B*a^2*b + 7*A*a*b^2)*c^2)*e^
3 + (105*B*a*c^4*d^2*e - 21*(4*B*a*b*c^3 - 5*A*a*c^4)*d*e^2 + (24*B*a*b^2*c
^2 - (25*B*a^2 + 28*A*a*b)*c^3)*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a*c^5*x
)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.18 $\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 528

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{e(10Bcd-4bBe+5Ace)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$
$$+\frac{(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{a}(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{15c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae))-\frac{\sqrt{c}(2aBe(5cd-2be)-5Ac(3cd^2-ae^2))}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/15*e*(5*A*c*e-4*B*b*e+10*B*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*B*e^2*x^3
*(c*x^4+b*x^2+a)^(1/2)/c+1/15*(10*A*c*e*(-b*e+3*c*d)+B*(15*c^2*d^2+8*b^2*e^
2-c*e*(9*a*e+20*b*d)))*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2)
)-1/15*a^(1/4)*(10*A*c*e*(-b*e+3*c*d)+B*(15*c^2*d^2+8*b^2*e^2-c*e*(9*a*e+20
*b*d)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2
))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(1
0*A*c*e*(-b*e+3*c*d)+B*(15*c^2*d^2+8*b^2*e^2-c*e*(9*a*e+20*b*d))-(2*a*B*e*(
-2*b*e+5*c*d)-5*A*c*(-a*e^2+3*c*d^2))*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1693, 1211, 1117, 1209}

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(-\frac{\sqrt{c}(2aBe(5cd-2be)-5Ac(3cd^2-ae}{\sqrt{a}}\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(B(-ce(9ae+20bd)+8b^2e^2+15c}{15c^{11/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{x\sqrt{a+bx^2+cx^4}(B(-ce(9ae+20bd)+8b^2e^2+15c^2d^2)+10Ace(3cd-be))}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{ex\sqrt{a+bx^2+cx^4}(5Ace-4bBe+10Bcd)}{15c^2}+\frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*(10*B*c*d - 4*b*B*e + 5*A*c*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*
e^2*x^3*Sqrt[a + b*x^2 + c*x^4])/(5*c) + ((10*A*c*e*(3*c*d - b*e) + B*(15*c
^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(15*
c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(10*A*c*e*(3*c*d - b*e) + B*(15
*c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*
x^4]) + (a^(1/4)*(10*A*c*e*(3*c*d - b*e) + B*(15*c^2*d^2 + 8*b^2*e^2 - c*e*
(20*b*d + 9*a*e)) - (Sqrt[c]*(2*a*B*e*(5*c*d - 2*b*e) - 5*A*c*(3*c*d^2 - a*
e^2)))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{5Acd^2+(5Bcd^2+10Acde-3aBe^2)x^2+e(10Bcd-4bBe+5Ace)x^4}{\sqrt{a+bx^2+cx^4}}\,dx}{5c} \\
&= \frac{e(10Bcd-4bBe+5Ace)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c} \\
&\quad + \frac{\int \frac{-2aBe(5cd-2be)+5Ac(3cd^2-ae^2)+(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^2} \\
&= \frac{e(10Bcd-4bBe+5Ace)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a+bx^2+cx^4}}{5c} \\
&\quad - \frac{\left(\sqrt{a}(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae)))\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^{5/2}} \\
&\quad + \frac{\left(\sqrt{a}\left(10Ace(3cd-be)+B(15c^2d^2+8b^2e^2-ce(20bd+9ae))+\frac{\sqrt{c}(-2aBe(5cd-2be)+5Ac(3cd^2-ae^2))}{\sqrt{a}}\right)\right)}{15c^{5/2}}
\end{aligned}$$

$$= \frac{e(10Bcd - 4bBe + 5Ace)x\sqrt{a + bx^2 + cx^4}}{15c^2} + \frac{Be^2x^3\sqrt{a + bx^2 + cx^4}}{5c}$$
$$+ \frac{(10Ace(3cd - be) + B(15c^2d^2 + 8b^2e^2 - ce(20bd + 9ae)))\,x\sqrt{a + bx^2 + cx^4}}{15c^{5/2}\left(\sqrt{a} + \sqrt{c}x^2\right)}$$
$$- \frac{\sqrt[4]{a}(10Ace(3cd - be) + B(15c^2d^2 + 8b^2e^2 - ce(20bd + 9ae)))\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\Big(2\,\mathrm{ta}}{15c^{11/4}\sqrt{a + bx^2 + cx^4}}$$
$$+ \frac{\sqrt[4]{a}\left(10Ace(3cd - be) + B(15c^2d^2 + 8b^2e^2 - ce(20bd + 9ae)) - \frac{\sqrt{c}(2aBe(5cd-2be)-5Ac(3cd^2-ae^2))}{\sqrt{a}}\right)\Big(}{30c^{11/4}\sqrt{a + bx^2 + cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.81 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.28

$$\int \frac{(A + Bx^2)(d + ex^2)^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$
$$= \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}ex(a + bx^2 + cx^4)(5Ace + B(10cd - 4be + 3cex^2)) + i\left(-b + \sqrt{b^2 - 4ac}\right)(10Ace(3cd - be)}{}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4)*(5*A*c*e + B*(
10*c*d - 4*b*e + 3*c*e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*(10*A*c*e*(3*c*d
- b*e) + B*(15*c^2*d^2 + 8*b^2*e^2 - c*e*(20*b*d + 9*a*e)))*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])] - I*(-8*b^3*B*e^2 + 2*b^2*e*(10*B*c*d + 5*A*c*e + 4*B*Sqrt[b^2 - 4*a*c
]*e) - b*c*(15*B*c*d^2 + B*e*(20*Sqrt[b^2 - 4*a*c]*d - 17*a*e) + 10*A*e*(3*
c*d + Sqrt[b^2 - 4*a*c]*e)) + c*(B*(-9*a*Sqrt[b^2 - 4*a*c]*e^2 + 5*c*d*(3*S
qrt[b^2 - 4*a*c]*d - 4*a*e)) + 10*A*c*(3*c*d^2 + e*(3*Sqrt[b^2 - 4*a*c]*d -
 a*e))))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 508, normalized size of antiderivative = 0.96

method	result
elliptic	$\frac{B e^2 x^3 \sqrt{c x^4+b x^2+a}}{5c} + \frac{\left(A e^2+2Bed-\frac{4bBe^2}{5c}\right) x\sqrt{c x^4+b x^2+a}}{3c} + \frac{\left(d^2 A-\frac{a\left(A e^2+2Bed-\frac{4bBe^2}{5c}\right)}{3c}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\cdots}{4\sqrt{\frac{-b+\sqrt{-\cdots}}{\cdots}}\cdots}$
risch	$\frac{ex(3Bex^2c+5Ace-4Bbe+10Bcd)\sqrt{c x^4+b x^2+a}}{15c^2} - \frac{15Ac^2d^2\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-\cdots}}{a}}}{2}\cdots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c x^4+b x^2+a}\cdots}$
default	Expression too large to display

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*e^2*x^3*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(A*e^2+2*B*e*d-4/5*b/c*B*e^2)/c*x
*(c*x^4+b*x^2+a)^(1/2)+1/4*(d^2*A-1/3*a/c*(A*e^2+2*B*e*d-4/5*b/c*B*e^2))*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellip
ticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))-1/2*(2*A*e*d+B*d^2-3/5*a/c*B*e^2-2/3*b/c*(A*e^2+2*
B*e*d-4/5*b/c*B*e^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+
(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt{\frac{1}{2}}\left((15\,Bac^3d^2 - 10\,(2\,Babc^2 - 3\,Aac^3)de + (8\,Bab^2c - (9\,Ba^2 + 10\,Aab)c^2)e^2)x\sqrt{\frac{b^2-4\,ac}{c^2}} - (15\,Babc^2d^2 \cdots\right.}{\cdots}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/30*(sqrt(1/2)*((15*B*a*c^3*d^2 - 10*(2*B*a*b*c^2 - 3*A*a*c^3)*d*e + (8*B*
a*b^2*c - (9*B*a^2 + 10*A*a*b)*c^2)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2) - (15*B*
a*b*c^2*d^2 - 10*(2*B*a*b^2*c - 3*A*a*b*c^2)*d*e + (8*B*a*b^3 - (9*B*a^2*b
+ 10*A*a*b^2)*c)*e^2)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*el
liptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(
b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((15*(B*a*c^3
 - A*c^4)*d^2 - 10*(2*B*a*b*c^2 - (3*A + B)*a*c^3)*d*e + (8*B*a*b^2*c + 5*A
*a*c^3 - (9*B*a^2 + 2*(5*A + 2*B)*a*b)*c^2)*e^2)*x*sqrt((b^2 - 4*a*c)/c^2)
- (15*(B*a*b*c^2 + A*b*c^3)*d^2 - 10*(2*B*a*b^2*c - (3*A - B)*a*b*c^2)*d*e
+ (8*B*a*b^3 - 5*A*a*b*c^2 - (9*B*a^2*b + 2*(5*A - 2*B)*a*b^2)*c)*e^2)*x)*s
qrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*
sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2
) + b^2 - 2*a*c)/(a*c)) + 2*(3*B*a*c^3*e^2*x^4 + 15*B*a*c^3*d^2 - 10*(2*B*a
*b*c^2 - 3*A*a*c^3)*d*e + (8*B*a*b^2*c - (9*B*a^2 + 10*A*a*b)*c^2)*e^2 + (1
0*B*a*c^3*d*e - (4*B*a*b*c^2 - 5*A*a*c^3)*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)
)/(a*c^4*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.19 $\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 368

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bex\sqrt{a+bx^2+cx^4}}{3c}+\frac{(3Bcd-2bBe+3Ace)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{\sqrt[4]{a}(3Bcd-2bBe+3Ace)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}\left(3Bcd-2bBe+3Ace+\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(\right.\right.}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/3*B*e*x*(c*x^4+b*x^2+a)^(1/2)/c+1/3*(3*A*c*e-2*B*b*e+3*B*c*d)*x*(c*x^4+b*
x^2+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-1/3*a^(1/4)*(3*A*c*e-2*B*b*e+3*B
*c*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))
^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(3*B*c
*d-2*B*b*e+3*A*c*e+(3*A*c*d-B*a*e)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/
2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1693, 1211, 1117, 1209}

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}+3Ace-2bBe+\right.}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(3Ace-2bBe+3Bcd)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}$$

$$+\frac{x\sqrt{a+bx^2+cx^4}(3Ace-2bBe+3Bcd)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{Bex\sqrt{a+bx^2+cx^4}}{3c}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*e*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + ((3*B*c*d - 2*b*B*e + 3*A*c*e)*x*Sq
rt[a + b*x^2 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(3*B*
c*d - 2*b*B*e + 3*A*c*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(3*B*c
*d - 2*b*B*e + 3*A*c*e + (Sqrt[c]*(3*A*c*d - a*B*e))/Sqrt[a])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[
a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{Bex\sqrt{a+bx^2+cx^4}}{3c} + \frac{\int \frac{3Acd-aBe+(3Bcd-2bBe+3Ace)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{Bex\sqrt{a+bx^2+cx^4}}{3c} - \frac{\left(\sqrt{a}(3Bcd-2bBe+3Ace)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} + \frac{\left(\sqrt{a}\left(3Bcd-2bBe+3Ace+\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}}$$

$$= \frac{Bex\sqrt{a+bx^2+cx^4}}{3c} + \frac{(3Bcd-2bBe+3Ace)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}(3Bcd-2bBe+3Ace)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(3Bcd-2bBe+3Ace+\frac{\sqrt{c}(3Acd-aBe)}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}(2}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.35 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.42

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{4Bc\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}ex(a+bx^2+cx^4) - i\left(-b+\sqrt{b^2-4ac}\right)(-3Bcd+2bBe-3Ace)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b}}{b-}}}{}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e*x*(a + b*x^2 + c*x^4) - I*(-b + Sq
rt[b^2 - 4*a*c])*(-3*B*c*d + 2*b*B*e - 3*A*c*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2
*b^2*B*e - c*(6*A*c*d + 3*B*Sqrt[b^2 - 4*a*c]*d - 2*a*B*e + 3*A*Sqrt[b^2 -
4*a*c]*e) + b*(3*B*c*d + 3*A*c*e + 2*B*Sqrt[b^2 - 4*a*c]*e))*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{Bex\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(Ad-\frac{aeB}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-\right.}}{ac}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$\frac{Bex\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\frac{3Acd\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}}{}$
default	$\frac{Ad\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+eB\left(\frac{x\sqrt{c\,x^4+b}}{3c}\right.$

```
[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*e*x*(c*x^4+b*x^2+a)^(1/2)/c+1/4*(A*d-1/3*a/c*e*B)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))-1/2*(A*e+B*d-2/3*b/c*e*B)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.12 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$=\frac{\sqrt{\frac{1}{2}}\left(\left(3\,Bac^2d-(2\,Babc-3\,Aac^2)e\right)x\sqrt{\frac{b^2-4\,ac}{c^2}}-\left(3\,Babcd-(2\,Bab^2-3\,Aabc)e\right)x\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\,E(a}{}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(1/2)*((3*B*a*c^2*d - (2*B*a*b*c - 3*A*a*c^2)*e)*x*sqrt((b^2 - 4*a
*c)/c^2) - (3*B*a*b*c*d - (2*B*a*b^2 - 3*A*a*b*c)*e)*x)*sqrt(c)*sqrt((c*sqr
t((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(
a*c)) - sqrt(1/2)*((3*(B*a*c^2 - A*c^3)*d - (2*B*a*b*c - (3*A + B)*a*c^2)*e
)*x*sqrt((b^2 - 4*a*c)/c^2) - (3*(B*a*b*c + A*b*c^2)*d - (2*B*a*b^2 - (3*A
- B)*a*b*c)*e)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_
f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqr
t((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + 2*(B*a*c^2*e*x^2 + 3*B*a*c^2*d
 - (2*B*a*b*c - 3*A*a*c^2)*e)*sqrt(c*x^4 + b*x^2 + a))/(a*c^3*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{\sqrt{cx^4+bx^2+a}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{(B\,x^2+A)\,(e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.20 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 283

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] B*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*B*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B+A*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1211, 1117, 1209}

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$+ \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\text{integral} = \left(A+\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx-\frac{\left(\sqrt{a}B\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$=\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{a}B+A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(B\left(-b+\sqrt{b^2-4ac}\right)E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(bB\right.}{2\sqrt{2}c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.28

method	result
default	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2(-b+}{}}}{}$
elliptic	$\frac{A\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}-\frac{Ba\sqrt{2}\,\sqrt{4-\frac{2(-b+}{}}}{}$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$=\frac{2\sqrt{cx^4+bx^2+a}Bac+\sqrt{\frac{1}{2}}\left(Bacx\sqrt{\frac{b^2-4ac}{c^2}}-Babx\right)\sqrt{c}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{b}}{}\right.}{}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(c*x^4 + b*x^2 + a)*B*a*c + sqrt(1/2)*(B*a*c*x*sqrt((b^2 - 4*a*c
)/c^2) - B*a*b*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_
e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqr
t((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*((B*a*c - A*c^2)*x*s
```

```
qrt((b^2 - 4*a*c)/c^2) - (B*a*b + A*b*c)*x)*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a
*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2
) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))/(a*c^
2*x)
```

Sympy [F]

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Maxima [F]

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx = \int \frac{B\,x^2 + A}{\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.21 $\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 436

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$
$$-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2\right.\right.}{4\sqrt[4]{c}de\left(cd^2-ae^2\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] -1/2*(-A*e+B*d)*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b
*x^2+a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(B*a^(1/2)-A
*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/a^(1/4)/c^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/4*a^(3/
4)*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)
+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)
+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2)
)^2)^(1/2)/c^(1/4)/d/e/(-a*e^2+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1722, 1117, 1720}

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{}{\sqrt{c}}\right.\right.}{4\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}\,(cd^2-ae^2)}$$

$$-\frac{(Bd-Ae)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

$$-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((B*d - A*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*S
qrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]) - (
(Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*
x^2 + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticP
i[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4)*d*e*(c*d^2 - a*e^2)*Sq
rt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
```

```
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\sqrt{a}B - A\sqrt{c}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d - \sqrt{a}e} + \frac{\left(\sqrt{a}(Bd - Ae)\right)\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}d - \sqrt{a}e}$$

$$= -\frac{(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2 - bde + ae^2}} - \frac{\left(\sqrt{a}B - A\sqrt{c}\right)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}d - \sqrt{a}e\right)\sqrt{a + bx^2 + cx^4}} + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right)(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{c}de\left(\sqrt{c}d - \sqrt{a}e\right)\sqrt{a + bx^2 + cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.68

$$\int \frac{A + Bx^2}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1 + \frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(Bd\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right), \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) + (-Bd + \ldots\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a + bx^2 + cx^4}}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*d*EllipticF[I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
a*c])] + (-(B*d) + A*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*A
rcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.82

method	result
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{cx^4+bx^2+a}}+\frac{(Ae-Bd)\sqrt{2}\,\sqrt{1+\frac{b\cdots}{2\cdots}}\cdots}{\cdots}$
elliptic	$\frac{B\sqrt{2}\,\sqrt{4+\frac{2b\,x^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,\sqrt{4+\frac{2b\,x^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,F\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}+\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^2}{2a}-\frac{x^2\cdots}{\cdots}}\cdots}{\cdots}$

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(A*e-B*d)/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+
b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2
*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Elliptic
Pi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2
))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.22 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 782

$$
\begin{aligned}
&\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\sqrt{a+bx^2+cx^4}}\,dx \\
&= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} \\
&\quad - \frac{\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\sqrt{e}\left(cd^2-bde+ae^2\right)^{3/2}} \\
&\quad - \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} \\
&\quad + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(B\left(cd^3-ade^2\right)-Ae\left(3cd^2-e(2bd-ae)\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\cdots}{4\sqrt{a}\cdots}\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

```
[Out] -1/4*(B*(-a*d*e^2+c*d^3)-A*e*(3*c*d^2-e*(-a*e+2*b*d)))*arctan(x*(a*e^2-b*d*
e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(3/2)/(a*e^2-b*d*e+
c*d^2)^(3/2)/e^(1/2)-1/2*e*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*
e+c*d^2)/(e*x^2+d)+1/2*(-A*e+B*d)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-
b*d*e+c*d^2)/(a^(1/2)+x^2*c^(1/2))-1/2*a^(1/4)*c^(1/4)*(-A*e+B*d)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d/(a*e^2-b*d
```

```
*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*A*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x
^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))
/(c*x^4+b*x^2+a)^(1/2)+1/8*(B*(-a*d*e^2+c*d^3)-A*e*(3*c*d^2-e*(-a*e+2*b*d))
)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^
2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(
1/4)/c^(1/4)/d^2/e/(a*e^2-b*d*e+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1710, 1728, 1209, 1722, 1117, 1720}

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt[4]{a}\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(Bd-Ae)E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}$$

$$+\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\left(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae))\right)\text{EllipticPi}\left(-\frac{(\sqrt{c}d-}{4\sqrt{a}}\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)(ae^2-bde+cd^2)}$$

$$-\frac{\left(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae))\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\sqrt{e}\,(ae^2-bde+cd^2)^{3/2}}$$

$$+\frac{A\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d\sqrt{a+bx^2+cx^4}\left(\sqrt{c}d-\sqrt{a}e\right)}$$

$$+\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\left(\sqrt{a}+\sqrt{c}x^2\right)(ae^2-bde+cd^2)}-\frac{ex\sqrt{a+bx^2+cx^4}(Bd-Ae)}{2d\,(d+ex^2)\,(ae^2-bde+cd^2)}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[c]*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2
)*(Sqrt[a] + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(2*d
*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) - ((B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^
2 - e*(2*b*d - a*e)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[
e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2)*Sqrt[e]*(c*d^2 - b*d*e + a*e^2)^(3
/2)) - (a^(1/4)*c^(1/4)*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
```

```
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^
2 + c*x^4]) + (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 +
 c*x^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*(c*d^3 - a*d*e^2) - A*e*(3*c*d^2 - e
*(2*b*d - a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c
]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(
1/4)*c^(1/4)*d^2*e*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a +
 b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
```

```
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-aBde-A\left(2cd^2-e(2bd-ae)\right)-2cd(Bd-Ae)x^2-ce(Bd-Ae)x^4}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(cd^2-bde+ae^2\right)} \\
&= -\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} - \frac{\int \frac{-\sqrt{a}c^{3/2}de(Bd-Ae)+ce\left(-aBde-A\left(2cd^2-e(2bd-ae)\right)\right)+\left(-2c^2de(Bd-Ae)+ce(Bd-Ae)\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx}{2cde\left(cd^2-bde+ae^2\right)} \\
&\quad - \frac{\left(\sqrt{a}\sqrt{c}(Bd-Ae)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{2d\left(cd^2-bde+ae^2\right)}
\end{aligned}$$

$$= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(\sqrt{a}+\sqrt{c}x^2)} - \frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(d+ex^2)} - \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{(A\sqrt{c})\int\frac{1}{\sqrt{a+bx^2+cx^4}}dx}{d(\sqrt{c}d-\sqrt{a}e)} + \frac{(\sqrt{a}(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae))))\int\frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx}{2d(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)}$$

$$= \frac{\sqrt{c}(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(\sqrt{a}+\sqrt{c}x^2)} - \frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)(d+ex^2)} - \frac{(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae)))\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\sqrt{e}(cd^2-bde+ae^2)^{3/2}} - \frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{A\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}d(\sqrt{c}d-\sqrt{a}e)\sqrt{a+bx^2+cx^4}} + \frac{(\sqrt{c}d+\sqrt{a}e)(B(cd^3-ade^2)-Ae(3cd^2-e(2bd-ae)))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(-\frac{(\sqrt{c}d-\sqrt{}}{4\sqrt{a}\sqrt{}}\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2e(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.38 (sec) , antiderivative size = 1853, normalized size of antiderivative = 2.37

$$\int\frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}dx = -\frac{i\left(-4i\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de^2(Bd-Ae)x(a+bx^2+cx^4)+\sqrt{2}B(b-\sqrt{b^2-4ac})d^2e\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2}{b-\sqrt{}}}\right.}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-1/8*I)*((-4*I)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*(B*d - A*e)*x*(a +
b*x^2 + c*x^4) + Sqrt[2]*B*(b - Sqrt[b^2 - 4*a*c])*d^2*e*Sqrt[(b + Sqrt[b^2
```

```
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - Ellipti
cF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])]) + Sqrt[2]*A*(-b + Sqrt[b^2 - 4*a*c])*d*e^2*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(EllipticE[I*ArcSinh[Sqrt[2]*S
qrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + 2*Sqrt[2]*B*c*d^3*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - 2*Sqrt[2]*A*c*d^2*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 2*Sqrt[2]*B*c*d^3*Sqrt[(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])] + 6*Sqrt[2]*A*c*d^2*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c])] - 4*Sqrt[2]*A*b*d*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d
+ e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])] + 2*Sqrt[2]*a*B*d*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2
)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] + 2*Sqrt[2]*a*A*e^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellipti
cPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*(c*d^3 + d*e*(-(b*d) + a*e))*(d + e*x^2)*S
qrt[a + b*x^2 + c*x^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1494 vs. 2(745) = 1490.

Time = 3.17 (sec) , antiderivative size = 1495, normalized size of antiderivative = 1.91

method	result	size
default	Expression too large to display	1495
elliptic	Expression too large to display	2301

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*
(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2
)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))+(A*e-B*d)/e*(1/2*e^2*x*(c*x^
4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/(a*e^2-b*d*e+c*d^2)*
2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^
2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/d/(a*e^2-b*d*e+c*d^2)*a*2^(
1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^
(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*c*e/d/(a*e^
2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2
*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^
(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2
*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1
/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
))*a-1/(a*e^2-b*d*e+c*d^2)*e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*
(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a
*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*b+3
/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b
*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c
+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*c)
```

Fricas [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c*e^2*x^8 + (2*c*d*e + b*e^2)
*x^6 + (c*d^2 + 2*b*d*e + a*e^2)*x^4 + a*d^2 + (b*d^2 + 2*a*d*e)*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}}\,dx=\int \frac{A+Bx^2}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}}\,dx=\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx=\int \frac{B\,x^2+A}{{\left(e\,x^2+d\right)}^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.23 $\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 1125

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$$
$$= -\frac{\sqrt{c}(3Ae(3cd^2-e(2bd-ae))-Bd(5cd^2-e(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2(cd^2-bde+ae^2)^2(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{4d(cd^2-bde+ae^2)(d+ex^2)^2}$$
$$+\frac{e(3Ae(3cd^2-e(2bd-ae))-Bd(5cd^2-e(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2(cd^2-bde+ae^2)^2(d+ex^2)}$$
$$-\frac{(Bd(3c^2d^4-10acd^2e^2+ae^3(4bd-ae))-Ae(15c^2d^4-2cd^2e(10bd-3ae)+e^2(8b^2d^2-8abde+3a^2e^2 \ldots}{16d^{5/2}\sqrt{e}(cd^2-bde+ae^2)^{5/2}}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}(3Ae(3cd^2-e(2bd-ae))-Bd(5cd^2-e(2bd+ae)))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\ldots\right.\right.}{8d^2(cd^2-bde+ae^2)^2\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{c}(\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)+4Ad(cd-be))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arcta\ldots\right.}{8\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{(\sqrt{c}d+\sqrt{a}e)(Bd(3c^2d^4-10acd^2e^2+ae^3(4bd-ae))-Ae(15c^2d^4-2cd^2e(10bd-3ae)+e^2(8b^2d^2-\ldots}{32\sqrt[4]{a}\sqrt[4]{c}d^3e(\sqrt{c}d-\sqrt{a}e)(cd^2\ldots}$$

```
[Out] -1/16*(B*d*(3*c^2*d^4-10*a*c*d^2*e^2+a*e^3*(-a*e+4*b*d))-A*e*(15*c^2*d^4-2*
c*d^2*e*(-3*a*e+10*b*d)+e^2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)))*arctan(x*(a*e
^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(5/2)/(a*e^2
```

```
-b*d*e+c*d^2)^(5/2)/e^(1/2)-1/4*e*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e
^2-b*d*e+c*d^2)/(e*x^2+d)^2+1/8*e*(3*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(5*c*
d^2-e*(a*e+2*b*d)))*x*(c*x^4+b*x^2+a)^(1/2)/d^2/(a*e^2-b*d*e+c*d^2)^2/(e*x^
2+d)-1/8*(3*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(5*c*d^2-e*(a*e+2*b*d)))*x*c^(
1/2)*(c*x^4+b*x^2+a)^(1/2)/d^2/(a*e^2-b*d*e+c*d^2)^2/(a^(1/2)+x^2*c^(1/2))+
1/8*a^(1/4)*c^(1/4)*(3*A*e*(3*c*d^2-e*(-a*e+2*b*d))-B*d*(5*c*d^2-e*(a*e+2*b
*d)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))
^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/d^2/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+a)^(1/2)+1/32*(B*d*(3*c^2*d^4-10*
a*c*d^2*e^2+a*e^3*(-a*e+4*b*d))-A*e*(15*c^2*d^4-2*c*d^2*e*(-3*a*e+10*b*d)+e
^2*(3*a^2*e^2-8*a*b*d*e+8*b^2*d^2)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(
1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)
/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^3/e/(a*e^2-b*d*e+c*d^2)^2
/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/8*c^(1/4)*(cos(2*arctan(c^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*
arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^
(1/2))*(a*e*(3*A*e+B*d)+4*A*d*(-b*e+c*d)+d*(-A*e+B*d)*a^(1/2)*c^(1/2))*((c*
x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*e^2-b*d*e+c*d^2)
/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used

$= \{1710, 1728, 1209, 1722, 1117, 1720\}$

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$$

$$= -\frac{\sqrt{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\sqrt{cx^4+bx^2+a}x}{8d^2(cd^2-bed+ae^2)^2(\sqrt{c}x^2+\sqrt{a})}$$
$$+\frac{e(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\sqrt{cx^4+bx^2+a}x}{8d^2(cd^2-bed+ae^2)^2(ex^2+d)}$$
$$-\frac{e(Bd-Ae)\sqrt{cx^4+bx^2+a}x}{4d(cd^2-bed+ae^2)(ex^2+d)^2}$$
$$-\frac{(B(3c^2d^5-10ace^2d^3+ae^3(4bd-ae)d)-Ae(15c^2d^4-2ce(10bd-3ae)d^2+e^2(8b^2d^2-8abed+3a^2e^2 \cdots}{16d^{5/2}\sqrt{e}(cd^2-bed+ae^2)^{5/2}}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\cdots\right.\right.}{8d^2(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+a}}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)+4Ad(cd-be)\right)(\sqrt{c}x^2+\sqrt{a})\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\text{EllipticF}\left(2\,\text{arcta}\cdots\right.}{8\sqrt[4]{a}d^2(\sqrt{c}d-\sqrt{a}e)(cd^2-bed+ae^2)\sqrt{cx^4+bx^2+a}}$$
$$+\frac{(\sqrt{c}d+\sqrt{a}e)(B(3c^2d^5-10ace^2d^3+ae^3(4bd-ae)d)-Ae(15c^2d^4-2ce(10bd-3ae)d^2+e^2(8b^2d^2-\cdots}{32\sqrt[4]{a}\sqrt[4]{c}d^3e(\sqrt{c}d-\sqrt{a}e)(cd^2 \cdots}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/8*(Sqrt[c]*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*c*d^3 - d*e*(2*b*d
+ a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(d^2*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a]
 + Sqrt[c]*x^2)) - (e*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4])/(4*d*(c*d^2 -
b*d*e + a*e^2)*(d + e*x^2)^2) + (e*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(
5*c*d^3 - d*e*(2*b*d + a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(8*d^2*(c*d^2 - b*
d*e + a*e^2)^2*(d + e*x^2)) - ((B*(3*c^2*d^5 - 10*a*c*d^3*e^2 + a*d*e^3*(4*
b*d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2*e*(10*b*d - 3*a*e) + e^2*(8*b^2*d^2
 - 8*a*b*d*e + 3*a^2*e^2)))*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]
*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(16*d^(5/2)*Sqrt[e]*(c*d^2 - b*d*e + a*
e^2)^(5/2)) + (a^(1/4)*c^(1/4)*(3*A*e*(3*c*d^2 - e*(2*b*d - a*e)) - B*(5*c*
d^3 - d*e*(2*b*d + a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(8*d^2*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*
x^4]) + (c^(1/4)*(Sqrt[a]*Sqrt[c]*d*(B*d - A*e) + a*e*(B*d + 3*A*e) + 4*A*d
*(c*d - b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*S
qrt[a + b*x^2 + c*x^4]) + ((Sqrt[c]*d + Sqrt[a]*e)*(B*(3*c^2*d^5 - 10*a*c*d
```

```
^3*e^2 + a*d*e^3*(4*b*d - a*e)) - A*e*(15*c^2*d^4 - 2*c*d^2*e*(10*b*d - 3*a
*e) + e^2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d
 - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(32*a^(1/4)*c^(1/4)*d^3*e*(Sqrt[c]*d - Sqrt[a]*e)*
(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\text{integral} = -\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{4d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2} - \frac{\int \frac{-4Acd^2-aBde+Ae(4bd-3ae)-2(Bd-Ae)(2cd-be)x^2+ce(Bd-Ae)x^4}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx}{4d\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{4d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2} + \frac{e(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)} + \frac{\int \frac{ade(Bd-Ae)(5cd-2be)+(4Acd^2+aBde-Ae(4bd-3ae))(2cd^2-e(2bd-ae))-2cd(Ae(8cd^2-e(5bd-2ae))-B(4cd^3-de(bd+2ae)}{(d+ex^2)\sqrt{a+bx^2+cx^4}}}{8d^2\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{4d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2}$$
$$+\frac{e(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}$$
$$+\frac{\int \frac{ce\left(ade(Bd-Ae)(5cd-2be)+\left(4Acd^2+aBde-Ae(4bd-3ae)\right)\left(2cd^2-e(2bd-ae)\right)\right)-\sqrt{a}c^{3/2}de\left(3Ae\left(3cd^2-e(2bd-ae)\right)-B\left(5cd^3-de\right.\right.}{(d+ex}}{8cd^2e\,(c}$$
$$+\frac{\left(\sqrt{a}\sqrt{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{8d^2\left(cd^2-bde+ae^2\right)^2}$$
$$= -\frac{\sqrt{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2\left(cd^2-bde+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{4d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2}$$
$$+\frac{e(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan\right.}{8d^2\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{c}\left(\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)+4Ad(cd-be)\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{4d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)}$$
$$+\frac{\left(\sqrt{a}(B(3c^2d^5-10acd^3e^2+ade^3(4bd-ae))-Ae(15c^2d^4-2cd^2e(10bd-3ae)+e^2(8b^2d^2-8ab\right.}{8d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)^2}$$

$$= -\frac{\sqrt{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2\left(cd^2-bde+ae^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$-\frac{e(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{4d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^2}$$
$$+\frac{e(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\,x\sqrt{a+bx^2+cx^4}}{8d^2\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}$$
$$-\frac{(B(3c^2d^5-10acd^3e^2+ade^3(4bd-ae))-Ae(15c^2d^4-2cd^2e(10bd-3ae)+e^2(8b^2d^2-8abde}{16d^{5/2}\sqrt{e}\left(cd^2-bde+ae^2\right)^{5/2}}$$
$$+\frac{\sqrt[4]{a}\sqrt[4]{c}(3Ae(3cd^2-e(2bd-ae))-B(5cd^3-de(2bd+ae)))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan\right.}{8d^2\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}d(Bd-Ae)+ae(Bd+3Ae)+4Ad(cd-be)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\right.}{8\sqrt[4]{a}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(B(3c^2d^5-10acd^3e^2+ade^3(4bd-ae))-Ae(15c^2d^4-2cd^2e(10bd-3ae)+e^2(8\right.}{32\sqrt[4]{a}\sqrt[4]{c}d^3e\left(\sqrt{c}d-\sqrt{a}e\right)(}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.91 (sec) , antiderivative size = 781, normalized size of antiderivative = 0.69

$$\int\frac{A+Bx^2}{\left(d+ex^2\right)^3\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{-\frac{4de^2x(a+bx^2+cx^4)\left(2d(Bd-Ae)(cd^2+e(-bd+ae))+\left(-3Ae(3cd^2+e(-2bd+ae))+B(5cd^3-de(2bd+ae))\right)(d+ex^2)\right)}{(d+ex^2)^2}-\frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac}+2c}{b+\sqrt{b^2-4ac}}}}{}}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*d*e^2*x*(a + b*x^2 + c*x^4)*(2*d*(B*d - A*e)*(c*d^2 + e*(-(b*d) + a*e)
) + (-3*A*e*(3*c*d^2 + e*(-2*b*d + a*e)) + B*(5*c*d^3 - d*e*(2*b*d + a*e)))
*(d + e*x^2)))/(d + e*x^2)^2 - (I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
((-b + Sqrt[b^2 - 4*a*c])*d*e*(3*A*e*(3*c*d^2 + e*(-2*b*d + a*e)) + B*(-5*c
*d^3 + d*e*(2*b*d + a*e)))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + d*(B*d*(
6*c^2*d^3 + c*d*e*(-5*b*d + 5*Sqrt[b^2 - 4*a*c]*d - 6*a*e) - (-b + Sqrt[b^2
 - 4*a*c])*e^2*(2*b*d + a*e)) - A*e*(14*c^2*d^3 - 3*(-b + Sqrt[b^2 - 4*a*c]
```

```
)*e^2*(2*b*d - a*e) + c*d*e*(-17*b*d + 9*Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*Ell
ipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + 2*(B*(-3*c^2*d^5 + 10*a*c*d^3*e^2 + a*
d*e^3*(-4*b*d + a*e)) + A*e*(15*c^2*d^4 + 2*c*d^2*e*(-10*b*d + 3*a*e) + e^2
*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)))*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/
(32*d^3*e*(c*d^2 + e*(-(b*d) + a*e))^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4475 vs. 2(1080) = 2160.

Time = 4.71 (sec) , antiderivative size = 4476, normalized size of antiderivative = 3.98

method	result	size
default	Expression too large to display	4476
elliptic	Expression too large to display	5671

```
[In] int((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e*(1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/
(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/
a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2)
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/d/(a*e^
2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2
*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1
/2))-1/4*c*e/d/(a*e^2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^
(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(
-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*Ellip
ticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*
(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2
)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*
EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b
^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)*e/d*2^(1/2)/(-b/a+1/a*(-4*a*
c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1
/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipt
icPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1
/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2))*b+3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(
```

1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2
/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e
/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2))*c)+(A*e-B*d)/e*(1/4*e^2/(a*e^2-b*d*e+c*d^2)/d*x*(c*x^4+b*x^2+a)^(1
/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-2*b*d*e+3*c*d^2)/(a*e^2-b*d*e+c*d^2)^2/d^2*x
*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)-1/32*c/(a*e^2-b*d*e+c*d^2)^2/d*2^(1/2)/(-b
/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(
1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*E
llipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))*a*e^2+1/8*c/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-b/
a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1
/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))*b*e-1/(a*e^2-b*d*e+c*d^2)^2/d^2*e^3*2^(1/2)/(-b
/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2
))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-
4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2))*a*b+3/4/(a*e^2-b*d*e+c*d^2)^2*e^2/d*2^(1/2)/(
-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1
/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+
(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*a*c-7/32*c^2/(a*e^2-b*d*e+c*d^2)^2*d*2^(1/2
)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/
2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+15/8/(a*e^2-b*d*e+c*d^2)^2*d*2^(1/2)/(-b/
a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2)
)^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4
*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2))*c^2+9/16*e*c^2/(a*e^2-b*d*e+c*d^2)^2*a*2^(1/2)
/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2
))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-3/16*e^3*c/(a*e^2-b
*d*e+c*d^2)^2/d^2*a^2*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^
2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/
2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))-9/16*e*c^2/(a*e^2-b*d*e+c*d^2)^2*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)
^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2
x^2/a(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2

```
))*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-3/8*e^2*c/(a*e^2-b*d*e+c*d^2)^2/d*a*2^(1/2
)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/
2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b+3/8*e^2*c/(a*e^2
-b*d*e+c*d^2)^2/d*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/
a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2)
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))*b+3/16*e^3*c/(a*e^2-b*d*e+c*d^2)^2/d^2*a^2*2^(1/2)/(-b/a+1/a*(-4*a*
c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x
^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+3/8/(a*e^2-b*d*e+c*d^2)^2/d^3*e^4*2^
(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+
b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*a^2+1/(a*e^2-b*d*e+c*d^2)^2*e^2/d*2^
(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+
b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*b^2-5/2/(a*e^2-b*d*e+c*d^2)^2*e*2^(1
/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^
2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b
*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2
/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))*b*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/((d + e*x**2)**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^3), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}(ex^2+d)^3}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^3\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^3\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((A + B*x^2)/((d + e*x^2)^3*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/((d + e*x^2)^3*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.24 $\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 859

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(Ac(b^2cd^3-2acd(cd^2-3ae^2)-abe(3cd^2+ae^2))+aB(ab^2e^3+2ace(3cd^2-}{}$$

$$+\frac{Be^3x\sqrt{a+bx^2+cx^4}}{3c^2}$$

$$+\frac{(aB(6c^3d^3-8b^3e^3-9c^2de(bd+6ae)+bce^2(18bd+29ae))+3Ac(2ab^2e^3+6ace(cd^2-ae^2)-bcd(cd^2+3a}{3ac^{5/2}(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{(aB(6c^3d^3-8b^3e^3-9c^2de(bd+6ae)+bce^2(18bd+29ae))+3Ac(2ab^2e^3+6ace(cd^2-ae^2)-bcd(cd^2+3}{3a^{3/4}c^{11/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{(3Ac^3d^3-5a^2Bce^3-3\sqrt{a}c^{5/2}d^2(Bd+3Ae)+ae(3cd-2be)(3Bcd-4bBe+3Ace)+3a^{3/2}\sqrt{c}e^2(9Bcd-}{6a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{11/4}\sqrt{a+}}$$

```
[Out] x*(A*c*(b^2*c*d^3-2*a*c*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2))+a*B*(a*b^
2*e^3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))-(a*B*(-b*e+2*c*d)*(c^
2*d^2+b^2*e^2-c*e*(3*a*e+b*d))+A*c*(a*b^2*e^3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*
d*(3*a*e^2+c*d^2)))*x^2)/a/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/3*B*e^3
*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/3*(a*B*(6*c^3*d^3-8*b^3*e^3-9*c^2*d*e*(6*a*e
+b*d)+b*c*e^2*(29*a*e+18*b*d))+3*A*c*(2*a*b^2*e^3+6*a*c*e*(-a*e^2+c*d^2)-b*
c*d*(3*a*e^2+c*d^2)))*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(5/2)/(-4*a*c+b^2)/(a^(1/
2)+x^2*c^(1/2))-1/3*(a*B*(6*c^3*d^3-8*b^3*e^3-9*c^2*d*e*(6*a*e+b*d)+b*c*e^2
*(29*a*e+18*b*d))+3*A*c*(2*a*b^2*e^3+6*a*c*e*(-a*e^2+c*d^2)-b*c*d*(3*a*e^2+
c*d^2)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/
```

```
2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^
(1/2)/a^(3/4)/c^(11/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-1/6*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(3*A*c^3*d^
3-5*a^2*B*c*e^3+a*e*(-2*b*e+3*c*d)*(3*A*c*e-4*B*b*e+3*B*c*d)-3*c^(5/2)*d^2*
(3*A*e+B*d)*a^(1/2)+3*a^(3/2)*e^2*(3*A*c*e-4*B*b*e+9*B*c*d)*c^(1/2))*(a^(1/
2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(
11/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1692, 1693, 1211, 1117, 1209}

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{Bx\sqrt{cx^4+bx^2+a}e^3}{3c^2}$$

$$-\frac{(aB(6c^3d^3-9c^2e(bd+6ae)d-8b^3e^3+bce^2(18bd+29ae))+3Ac(2ab^2e^3+6ac(cd^2-ae^2)e-bcd(cd^2+}{3a^{3/4}c^{11/4}(b^2-4ac)\sqrt{cx^4+bx^2+}$$

$$-\frac{(3Ac^3d^3-3\sqrt{a}c^{5/2}(Bd+3Ae)d^2-5a^2Bce^3+ae(3cd-2be)(3Bcd-4bBe+3Ace)+3a^{3/2}\sqrt{c}e^2(9Bcd}{6a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{11/4}\sqrt{cx}$$

$$+\frac{(aB(6c^3d^3-9c^2e(bd+6ae)d-8b^3e^3+bce^2(18bd+29ae))+3Ac(2ab^2e^3+6ac(cd^2-ae^2)e-bcd(cd^2+}{3ac^{5/2}(b^2-4ac)\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

$$+\frac{x(-((aB(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))+Ac(ab^2e^3+2ac(3cd^2-ae^2)e-bcd(cd^2+3ae^2)))x^2)}{ac^2(b^2-4ac)\sqrt{}}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x]

[Out] (x*(A*c*(b^2*c*d^3 - 2*a*c*d*(c*d^2 - 3*a*e^2) - a*b*e*(3*c*d^2 + a*e^2)) +
 a*B*(a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)) - (a
*B*(2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)) + A*c*(a*b^2*e^3 +
 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2)))*x^2))/(a*c^2*(b^2 -
4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (B*e^3*x*Sqrt[a + b*x^2 + c*x^4])/(3*c^2)
 + ((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d + 6*a*e) + b*c*e^2*(18*b*d
 + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2 - a*e^2) - b*c*d*(c*d^2 +
 3*a*e^2)))*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*c^(5/2)*(b^2 - 4*a*c)*(Sqrt[a]
+ Sqrt[c]*x^2)) - ((a*B*(6*c^3*d^3 - 8*b^3*e^3 - 9*c^2*d*e*(b*d + 6*a*e) +
b*c*e^2*(18*b*d + 29*a*e)) + 3*A*c*(2*a*b^2*e^3 + 6*a*c*e*(c*d^2 - a*e^2) -
 b*c*d*(c*d^2 + 3*a*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b
/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(3/4)*c^(11/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 +
```

```
c*x^4]) - ((3*A*c^3*d^3 - 5*a^2*B*c*e^3 - 3*Sqrt[a]*c^(5/2)*d^2*(B*d + 3*A*
e) + a*e*(3*c*d - 2*b*e)*(3*B*c*d - 4*b*B*e + 3*A*c*e) + 3*a^(3/2)*Sqrt[c]*
e^2*(9*B*c*d - 4*b*B*e + 3*A*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(11/4)
*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
```

```
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

integral

$$= \frac{x(Ac(b^2cd^3 - 2acd(cd^2 - 3ae^2) - abe(3cd^2 + ae^2)) + aB(ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3ae^2)) -}{ac^2(b^2 - 4ac)\sqrt{a}} - \frac{\int \frac{\frac{a\left(ab^2Be^3 - bc\left(Bcd^3 + 3Acd^2e + 3aBde^2 + aAe^3\right) + 2c\left(aBe\left(3cd^2 - ae^2\right) + Acd\left(cd^2 + 3ae^2\right)\right)\right)}{c^2} - \frac{\left(aB\left(2c^3d^3 - 2b^3e^3 - 3c^2de(bd + 6ae) + bce^2(6bd + 7ae)\right) + Ac\left(2\right.}{c^2}}{\sqrt{a + bx^2 + cx^4}}}{a(b^2 - 4ac)}$$

$$= \frac{x(Ac(b^2cd^3 - 2acd(cd^2 - 3ae^2) - abe(3cd^2 + ae^2)) + aB(ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3}{ac^2(b^2 - 4} + \frac{Be^3x\sqrt{a + bx^2 + cx^4}}{3c^2} - \frac{\int \frac{\frac{a\left(4ab^2Be^3 - 3bc\left(Bcd^3 + 3Acd^2e + 3aBde^2 + aAe^3\right) + 2c\left(aBe\left(9cd^2 - 5ae^2\right) + 3Acd\left(cd^2 + 3ae^2\right)\right)\right)}{c} - \frac{\left(aB\left(6c^3d^3 - 8b^3e^3 - 9c^2de(bd + 6ae) + bce^2(1\right.\right.}{}}{\sqrt{a + bx^2 + cx^4}}}{3ac(b^2 - 4ac)}$$

$$= \frac{x(Ac(b^2cd^3 - 2acd(cd^2 - 3ae^2) - abe(3cd^2 + ae^2)) + aB(ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3}{ac^2(b^2 - 4} + \frac{Be^3x\sqrt{a + bx^2 + cx^4}}{3c^2} - \frac{(3Ac^3d^3 - 5a^2Bce^3 - 3\sqrt{a}c^{5/2}d^2(Bd + 3Ae) + ae(3cd - 2be)(3Bcd - 4bBe + 3Ace) + 3a^{3/2}\sqrt{}}{3\sqrt{a}\left(b - 2\sqrt{a}\sqrt{c}\right)c^{5/2}} - \frac{(aB(6c^3d^3 - 8b^3e^3 - 9c^2de(bd + 6ae) + bce^2(18bd + 29ae)) + 3Ac(2ab^2e^3 + 6ace(cd^2 - ae^2) -}{3\sqrt{a}c^{5/2}(b^2 - 4ac)}$$

$$= \frac{x(Ac(b^2cd^3 - 2acd(cd^2 - 3ae^2) - abe(3cd^2 + ae^2)) + aB(ab^2e^3 + 2ace(3cd^2 - ae^2) - bcd(cd^2 + 3a}{ac^2\left(b^2 - 4a\right.}$$

$$+ \frac{Be^3x\sqrt{a + bx^2 + cx^4}}{3c^2}$$

$$+ \frac{(aB(6c^3d^3 - 8b^3e^3 - 9c^2de(bd + 6ae) + bce^2(18bd + 29ae)) + 3Ac(2ab^2e^3 + 6ace(cd^2 - ae^2) - b}{3ac^{5/2}\left(b^2 - 4ac\right)\left(\sqrt{a} + \sqrt{c}x^2\right)}$$

$$- \frac{(aB(6c^3d^3 - 8b^3e^3 - 9c^2de(bd + 6ae) + bce^2(18bd + 29ae)) + 3Ac(2ab^2e^3 + 6ace(cd^2 - ae^2) - b}{3a^{3/4}c^{11/4}\left(b^2 - 4ac\right)\sqrt{a +}}$$

$$- \frac{\left(3Ac^3d^3 - 5a^2Bce^3 - 3\sqrt{a}c^{5/2}d^2(Bd + 3Ae) + ae(3cd - 2be)(3Bcd - 4bBe + 3Ace) + 3a^{3/2}\sqrt{c}e\right.}{6a^{3/4}\left(b - 2\sqrt{a}\sqrt{c}\right)c^{11/4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.27 (sec) , antiderivative size = 1058, normalized size of antiderivative = 1.23

$$\int \frac{(A + Bx^2)(d + ex^2)^3}{(a + bx^2 + cx^4)^{3/2}}\,dx = \frac{-4c\sqrt{\frac{c}{b + \sqrt{b^2 - 4ac}}}x(3Ac(b^2(cd^3 - ae^3x^2) + b(-a^2e^3 + c^2d^3x^2 - 3acde(d - ex^2))}{}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(3*A*c*(b^2*(c*d^3 - a*e^3*x^2) + b
*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e
*x^2) - c*d^2*(d + 3*e*x^2))) + a*B*(4*b^3*e^3*x^2 + b^2*e^2*(4*a*e - 9*c*d
*x^2 + c*e*x^4) - b*c*(3*c*d^2*(d - 3*e*x^2) + a*e^2*(9*d + 13*e*x^2)) - 2*
c*(5*a^2*e^3 + 3*c^2*d^3*x^2 + a*c*e*(-9*d^2 - 9*d*e*x^2 + 2*e^2*x^4)))) +
I*(-b + Sqrt[b^2 - 4*a*c])*(a*B*(-6*c^3*d^3 + 8*b^3*e^3 + 9*c^2*d*e*(b*d +
6*a*e) - b*c*e^2*(18*b*d + 29*a*e)) + 3*A*c*(-2*a*b^2*e^3 + 6*a*c*e*(-(c*d^
2) + a*e^2) + b*c*d*(c*d^2 + 3*a*e^2)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(a*B*(8*b^3
*(-b + Sqrt[b^2 - 4*a*c])*e^3 - 6*c^3*d^2*(Sqrt[b^2 - 4*a*c]*d - 6*a*e) + b
*c*e^2*(18*b^2*d - 18*b*Sqrt[b^2 - 4*a*c]*d + 37*a*b*e - 29*a*Sqrt[b^2 - 4*
a*c]*e) + c^2*e*(-9*b^2*d^2 + 2*a*e*(27*Sqrt[b^2 - 4*a*c]*d - 10*a*e) + 9*b
*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e))) + 3*A*c*(2*a*b^3*e^3 - b^2*(c^2*d^3 + 3*
a*c*d*e^2 + 2*a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*Sqrt[b^2 - 4*a*c]*d^3 + a*e
^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 2*a*c*(2*c^2*d^3 + 3*a*Sqrt[b^2 - 4*a
```

```
*c]*e^3 - 3*c*d*e*(Sqrt[b^2 - 4*a*c]*d - 2*a*e))))*Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12
*a*c^3*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^
4])
```

Maple [A] (verified)

Time = 12.57 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.33

method	result	size
elliptic	Expression too large to display	1141
default	Expression too large to display	2445
risch	Expression too large to display	2482

```
[In] int((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*c*(1/2/c^3*(2*A*a^2*c^2*e^3-A*a*b^2*c*e^3+3*A*a*b*c^2*d*e^2-6*A*a*c^3*d^
2*e+A*b*c^3*d^3-3*B*a^2*b*c*e^3+6*B*a^2*c^2*d*e^2+B*a*b^3*e^3-3*B*a*b^2*c*d
*e^2+3*B*a*b*c^2*d^2*e-2*B*a*c^3*d^3)/a/(4*a*c-b^2)*x^3-1/2/c^3*(A*a^2*b*c*
e^3-6*A*a^2*c^2*d*e^2+3*A*a*b*c^2*d^2*e+2*A*a*c^3*d^3-A*b^2*c^2*d^3+2*B*a^3
*c*e^3-B*a^2*b^2*e^3+3*B*a^2*b*c*d*e^2-6*B*a^2*c^2*d^2*e+B*a*b*c^2*d^3)/a/(
4*a*c-b^2)*x)/((x^4+1/c*b*x^2+a/c)*c)^(1/2)+1/3*B*e^3*x*(c*x^4+b*x^2+a)^(1/
2)/c^2+1/4*(-e*(A*b*c*e^2-3*A*c^2*d*e+B*a*c*e^2-B*b^2*e^2+3*B*b*c*d*e-3*B*c
^2*d^2)/c^3+1/c^3*(A*a*b*c*e^3-3*A*a*c^2*d*e^2+A*c^3*d^3+B*a^2*c*e^3-B*a*b^
2*e^3+3*B*a*b*c*d*e^2-3*B*a*c^2*d^2*e)/a-1/c^2*(A*a^2*b*c*e^3-6*A*a^2*c^2*d
*e^2+3*A*a*b*c^2*d^2*e+2*A*a*c^3*d^3-A*b^2*c^2*d^3+2*B*a^3*c*e^3-B*a^2*b^2*
e^3+3*B*a^2*b*c*d*e^2-6*B*a^2*c^2*d^2*e+B*a*b*c^2*d^3)/a/(4*a*c-b^2)-1/3*B*
e^3/c^2*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(1/c^2*e^2*(A*c*e-B*b*e+3*B*c*d)+
1/c^2*(2*A*a^2*c^2*e^3-A*a*b^2*c*e^3+3*A*a*b*c^2*d*e^2-6*A*a*c^3*d^2*e+A*b*
c^3*d^3-3*B*a^2*b*c*e^3+6*B*a^2*c^2*d*e^2+B*a*b^3*e^3-3*B*a*b^2*c*d*e^2+3*B
*a*b*c^2*d^2*e-2*B*a*c^3*d^3)/a/(4*a*c-b^2)-2/3*B/c^2*e^3*b)*a*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)
^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2623 vs. 2(841) = 1682.

Time = 0.13 (sec) , antiderivative size = 2623, normalized size of antiderivative = 3.05

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(sqrt(1/2)*((3*(2*B*a*b - A*b^2)*c^4*d^3 - 9*(B*a*b^2*c^3 - 2*A*a*b*c^
4)*d^2*e + 9*(2*B*a*b^3*c^2 - (6*B*a^2*b + A*a*b^2)*c^3)*d*e^2 - (8*B*a*b^4
*c + 18*A*a^2*b*c^3 - (29*B*a^2*b^2 + 6*A*a*b^3)*c^2)*e^3)*x^5 + (3*(2*B*a*
b^2 - A*b^3)*c^3*d^3 - 9*(B*a*b^3*c^2 - 2*A*a*b^2*c^3)*d^2*e + 9*(2*B*a*b^4
*c - (6*B*a^2*b^2 + A*a*b^3)*c^2)*d*e^2 - (8*B*a*b^5 + 18*A*a^2*b^2*c^2 - (
29*B*a^2*b^3 + 6*A*a*b^4)*c)*e^3)*x^3 + (3*(2*B*a^2*b - A*a*b^2)*c^3*d^3 -
9*(B*a^2*b^2*c^2 - 2*A*a^2*b*c^3)*d^2*e + 9*(2*B*a^2*b^3*c - (6*B*a^3*b + A
*a^2*b^2)*c^2)*d*e^2 - (8*B*a^2*b^4 + 18*A*a^3*b*c^2 - (29*B*a^3*b^2 + 6*A*
a^2*b^3)*c)*e^3)*x - ((3*(2*B*a - A*b)*c^5*d^3 - 9*(B*a*b*c^4 - 2*A*a*c^5)*
d^2*e + 9*(2*B*a*b^2*c^3 - (6*B*a^2 + A*a*b)*c^4)*d*e^2 - (8*B*a*b^3*c^2 +
18*A*a^2*c^4 - (29*B*a^2*b + 6*A*a*b^2)*c^3)*e^3)*x^5 + (3*(2*B*a*b - A*b^2
)*c^4*d^3 - 9*(B*a*b^2*c^3 - 2*A*a*b*c^4)*d^2*e + 9*(2*B*a*b^3*c^2 - (6*B*a
^2*b + A*a*b^2)*c^3)*d*e^2 - (8*B*a*b^4*c + 18*A*a^2*b*c^3 - (29*B*a^2*b^2
+ 6*A*a*b^3)*c^2)*e^3)*x^3 + (3*(2*B*a^2 - A*a*b)*c^4*d^3 - 9*(B*a^2*b*c^3
- 2*A*a^2*c^4)*d^2*e + 9*(2*B*a^2*b^2*c^2 - (6*B*a^3 + A*a^2*b)*c^3)*d*e^2
- (8*B*a^2*b^3*c + 18*A*a^3*c^3 - (29*B*a^3*b + 6*A*a^2*b^2)*c^2)*e^3)*x)*s
qrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*ell
iptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b
*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) + sqrt(1/2)*((3*(2*A*b*c^5
 - (2*B*a*b - (A - B)*b^2)*c^4)*d^3 + 9*(B*a*b^2*c^3 - (2*(A - B)*a*b + A*b
^2)*c^4)*d^2*e - 9*(2*B*a*b^3*c^2 - 2*A*a*b*c^4 - (6*B*a^2*b + (A - B)*a*b^
2)*c^3)*d*e^2 + (8*B*a*b^4*c + (2*(9*A - 5*B)*a^2*b - 3*A*a*b^2)*c^3 - (29*
B*a^2*b^2 + 2*(3*A - 2*B)*a*b^3)*c^2)*e^3)*x^5 + (3*(2*A*b^2*c^4 - (2*B*a*b
^2 - (A - B)*b^3)*c^3)*d^3 + 9*(B*a*b^3*c^2 - (2*(A - B)*a*b^2 + A*b^3)*c^3
)*d^2*e - 9*(2*B*a*b^4*c - 2*A*a*b^2*c^3 - (6*B*a^2*b^2 + (A - B)*a*b^3)*c^
2)*d*e^2 + (8*B*a*b^5 + (2*(9*A - 5*B)*a^2*b^2 - 3*A*a*b^3)*c^2 - (29*B*a^2
*b^3 + 2*(3*A - 2*B)*a*b^4)*c)*e^3)*x^3 + (3*(2*A*a*b*c^4 - (2*B*a^2*b - (A
 - B)*a*b^2)*c^3)*d^3 + 9*(B*a^2*b^2*c^2 - (2*(A - B)*a^2*b + A*a*b^2)*c^3)
*d^2*e - 9*(2*B*a^2*b^3*c - 2*A*a^2*b*c^3 - (6*B*a^3*b + (A - B)*a^2*b^2)*c
^2)*d*e^2 + (8*B*a^2*b^4 + (2*(9*A - 5*B)*a^3*b - 3*A*a^2*b^2)*c^2 - (29*B*
a^3*b^2 + 2*(3*A - 2*B)*a^2*b^3)*c)*e^3)*x + ((3*(2*A*c^6 + (2*B*a - (A + B
)*b)*c^5)*d^3 - 9*(B*a*b*c^4 - (2*(A + B)*a - A*b)*c^5)*d^2*e + 9*(2*B*a*b^
2*c^3 + 2*A*a*c^5 - (6*B*a^2 + (A + B)*a*b)*c^4)*d*e^2 - (8*B*a*b^3*c^2 + (
2*(9*A + 5*B)*a^2 + 3*A*a*b)*c^4 - (29*B*a^2*b + 2*(3*A + 2*B)*a*b^2)*c^3)*
```

```
e^3)*x^5 + (3*(2*A*b*c^5 + (2*B*a*b - (A + B)*b^2)*c^4)*d^3 - 9*(B*a*b^2*c^
3 - (2*(A + B)*a*b - A*b^2)*c^4)*d^2*e + 9*(2*B*a*b^3*c^2 + 2*A*a*b*c^4 - (
6*B*a^2*b + (A + B)*a*b^2)*c^3)*d*e^2 - (8*B*a*b^4*c + (2*(9*A + 5*B)*a^2*b
 + 3*A*a*b^2)*c^3 - (29*B*a^2*b^2 + 2*(3*A + 2*B)*a*b^3)*c^2)*e^3)*x^3 + (3
*(2*A*a*c^5 + (2*B*a^2 - (A + B)*a*b)*c^4)*d^3 - 9*(B*a^2*b*c^3 - (2*(A + B
)*a^2 - A*a*b)*c^4)*d^2*e + 9*(2*B*a^2*b^2*c^2 + 2*A*a^2*c^4 - (6*B*a^3 + (
A + B)*a^2*b)*c^3)*d*e^2 - (8*B*a^2*b^3*c + (2*(9*A + 5*B)*a^3 + 3*A*a^2*b)
*c^3 - (29*B*a^3*b + 2*(3*A + 2*B)*a^2*b^2)*c^2)*e^3)*x)*sqrt((b^2 - 4*a*c)
/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_f(arcsin(sq
rt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*
a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*((B*a*b^2*c^3 - 4*B*a^2*c^4)*e^3*x^6 +
3*(2*B*a^2 - A*a*b)*c^4*d^3 + (9*(B*a*b^2*c^3 - 4*B*a^2*c^4)*d*e^2 - (4*B*a
*b^3*c^2 + 12*A*a^2*c^4 - (16*B*a^2*b + 3*A*a*b^2)*c^3)*e^3)*x^4 - 9*(B*a^2
*b*c^3 - 2*A*a^2*c^4)*d^2*e + 9*(2*B*a^2*b^2*c^2 - (6*B*a^3 + A*a^2*b)*c^3)
*d*e^2 - (8*B*a^2*b^3*c + 18*A*a^3*c^3 - (29*B*a^3*b + 6*A*a^2*b^2)*c^2)*e^
3 + (3*(B*a*b*c^4 - 2*A*a*c^5)*d^3 - 9*(B*a*b^2*c^3 - (2*B*a^2 + A*a*b)*c^4
)*d^2*e + 9*(2*B*a*b^3*c^2 + 2*A*a^2*c^4 - (7*B*a^2*b + A*a*b^2)*c^3)*d*e^2
 - (8*B*a*b^4*c + (10*B*a^3 + 21*A*a^2*b)*c^3 - 3*(11*B*a^2*b^2 + 2*A*a*b^3
)*c^2)*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^5 - 4*a^2*c^6)*x^5 + (a
*b^3*c^4 - 4*a^2*b*c^5)*x^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**3/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^3}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^3/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^3/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)\,(d+ex^2)^3}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^3}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^3)/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.25 $\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 628

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x(aB(bcd^2-4acde+abe^2)-Ac(b^2d^2-2abde-2a(cd^2-ae^2))-(Ac(bcd^2-4acde+abe^2)-aB(2c^2d^2 \ldots}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Ac(bcd^2-4acde+abe^2)-2aB(c^2d^2+b^2e^2-ce(bd+3ae)))x\sqrt{a+bx^2+cx^4}}{ac^{3/2}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)}$$
$$+\frac{(Ac(bcd^2-4acde+abe^2)-2aB(c^2d^2+b^2e^2-ce(bd+3ae)))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\ldots\right.\right.}{a^{3/4}c^{7/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Ac^2d^2+3a^{3/2}B\sqrt{c}e^2-\sqrt{a}c^{3/2}d(Bd+2Ae)+ae(2Bcd-2bBe+Ace))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{Elli}\ldots}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})c^{7/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -x*(a*B*(a*b*e^2-4*a*c*d*e+b*c*d^2)-A*c*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^
2))-(A*c*(a*b*e^2-4*a*c*d*e+b*c*d^2)-a*B*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d)
))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-(A*c*(a*b*e^2-4*a*c*d*e+b*c*
d^2)-2*a*B*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d)))*x*(c*x^4+b*x^2+a)^(1/2)/a/c^(
3/2)/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))+(A*c*(a*b*e^2-4*a*c*d*e+b*c*d^2)-2*
a*B*(c^2*d^2+b^2*e^2-c*e*(3*a*e+b*d)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^
2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(7/4)/(-4*a*c+b^2)/(c*x^4+b*x
^2+a)^(1/2)-1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2
```

```
)/c^(1/2))^(1/2))*(A*c^2*d^2+a*e*(A*c*e-2*B*b*e+2*B*c*d)-c^(3/2)*d*(2*A*e+B
*d)*a^(1/2)+3*a^(3/2)*B*e^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)
/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(7/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^
4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 633, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1692, 1211, 1117, 1209}

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}dx = \frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(Ac(abe^2-4a}{a^{3/4}c^{7/4}(b^2-4ac)\sqrt{a+bx^2+c}} - \frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(3a^{3/2}B\sqrt{c}e^2+ae(Ace-2bBe+}{2a^{3/4}c^{7/4}(b-2\sqrt{a}\sqrt{c})\sqrt{a+bx^2+cx^4}} - \frac{x\left(c\left(\frac{aB(abe^2-4acde+bcd^2)}{c}-A(-2abde-2a(cd^2-ae^2)+b^2d^2)\right)-x^2(Ac(abe^2-4acde+bcd^2)-aB(-2ce(}{ac(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{x\sqrt{a+bx^2+cx^4}(Ac(abe^2-4acde+bcd^2)-2aB(-ce(3ae+bd)+b^2e^2+c^2d^2))}{ac^{3/2}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(c*((a*B*(b*c*d^2 - 4*a*c*d*e + a*b*e^2))/c - A*(b^2*d^2 - 2*a*b*d*e -
 2*a*(c*d^2 - a*e^2))) - (A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - a*B*(2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)))*x^2))/(a*c*(b^2 - 4*a*c)*Sqrt[a + b*x^2
 + c*x^4])) - ((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B*(c^2*d^2 + b^2*
e^2 - c*e*(b*d + 3*a*e)))*x*Sqrt[a + b*x^2 + c*x^4])/(a*c^(3/2)*(b^2 - 4*a*
c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2) - 2*a*B
*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*c^(7/4)*(b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4]) - ((A*c^2*d^2 + 3*a^(3/2)*B*Sqrt[c]*e^2 - Sqrt[a]*c^(3/
2)*d*(B*d + 2*A*e) + a*e*(2*B*c*d - 2*b*B*e + A*c*e))*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a
]*Sqrt[c])*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

integral =

$$-\frac{x\left(c\left(\frac{aB\left(bcd^2-4acde+abe^2\right)}{c}-A(b^2d^2-2abde-2a(cd^2-ae^2))\right)-(Ac(bcd^2-4acde+abe^2)-aB(2c^2d^2-\right.}{ac\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\int\frac{\frac{a(ae(4Bcd-bBe+2Ace)+cd(2Acd-b(Bd+2Ae)))}{c}+\frac{\left(Ac\left(bcd^2-4acde+abe^2\right)-2aB\left(c^2d^2+b^2e^2-ce(bd+3ae)\right)\right)x^2}{c}}{\sqrt{a+bx^2+cx^4}}dx}{a\left(b^2-4ac\right)}$$

$$= -\frac{x\left(c\left(\frac{aB(bcd^2-4acde+abe^2)}{c} - A(b^2d^2 - 2abde - 2a(cd^2 - ae^2))\right) - (Ac(bcd^2 - 4acde + abe^2) - aB(2\right.}{ac\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\left(Ac^2d^2 + 3a^{3/2}B\sqrt{c}e^2 - \sqrt{a}c^{3/2}d(Bd+2Ae) + ae(2Bcd - 2bBe + Ace)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b-2\sqrt{a}\sqrt{c}\right)c^{3/2}}$$
$$+ \frac{(Ac(bcd^2 - 4acde + abe^2) - 2aB(c^2d^2 + b^2e^2 - ce(bd+3ae)))\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}c^{3/2}\,(b^2-4ac)}$$
$$= -\frac{x\left(c\left(\frac{aB(bcd^2-4acde+abe^2)}{c} - A(b^2d^2 - 2abde - 2a(cd^2 - ae^2))\right) - (Ac(bcd^2 - 4acde + abe^2) - aB(2\right.}{ac\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$- \frac{(Ac(bcd^2 - 4acde + abe^2) - 2aB(c^2d^2 + b^2e^2 - ce(bd+3ae)))\,x\sqrt{a+bx^2+cx^4}}{ac^{3/2}\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+ \frac{(Ac(bcd^2 - 4acde + abe^2) - 2aB(c^2d^2 + b^2e^2 - ce(bd+3ae)))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\,\mathrm{ta}\right.}{a^{3/4}c^{7/4}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$- \frac{\left(Ac^2d^2 + 3a^{3/2}B\sqrt{c}e^2 - \sqrt{a}c^{3/2}d(Bd+2Ae) + ae(2Bcd - 2bBe + Ace)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{\right.}}}}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.99 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(-aB(abe^2 + 2c^2d^2x^2 + b^2e^2x^2 + bcd(d-2ex^2) - 2ace(2d+}{}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-(a*B*(a*b*e^2 + 2*c^2*d^2*x^2 + b
^2*e^2*x^2 + b*c*d*(d - 2*e*x^2) - 2*a*c*e*(2*d + e*x^2))) + A*c*(b^2*d^2 +
 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(d + 2*e*x^2))) -
 I*(-b + Sqrt[b^2 - 4*a*c])*(-(A*c*(b*c*d^2 - 4*a*c*d*e + a*b*e^2)) + 2*a*B
*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(2*a*B*(b
^2*(-b + Sqrt[b^2 - 4*a*c])*e^2 + c^2*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + c*e
```

```
*(b^2*d - b*Sqrt[b^2 - 4*a*c]*d + 4*a*b*e - 3*a*Sqrt[b^2 - 4*a*c]*e)) + A*c
*(b^2*(c*d^2 + a*e^2) - b*Sqrt[b^2 - 4*a*c]*(c*d^2 + a*e^2) - 4*a*c*(c*d^2
- Sqrt[b^2 - 4*a*c]*d*e + a*e^2)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]])/(4*a*c^2*(-b^2 + 4*
a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.27

method	result
elliptic	$-\frac{2c\left(\frac{\left(Aabc\,e^{2}-4Aa\,c^{2}de+Ab\,c^{2}d^{2}+2B\,a^{2}c\,e^{2}-Ba\,b^{2}e^{2}+2Babcde-2Ba\,c^{2}d^{2}\right)x^{3}}{2c^{2}a\left(4ac-b^{2}\right)}+\frac{\left(2A\,a^{2}c\,e^{2}-2Aabcde-2Aa\,c^{2}d^{2}+A\,b^{2}c\,d^{2}-B\,a^{2}b\,e^{2}+4B\,a^{2}\right.}{2c^{2}\left(4ac-b^{2}\right)a}\right.}{\sqrt{\left(x^{4}+\frac{b\,x^{2}}{c}+\frac{a}{c}\right)c}}$
default	Expression too large to display

```
[In] int((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/2/c^2*(A*a*b*c*e^2-4*A*a*c^2*d*e+A*b*c^2*d^2+2*B*a^2*c*e^2-B*a*b^2*
e^2+2*B*a*b*c*d*e-2*B*a*c^2*d^2)/a/(4*a*c-b^2)*x^3+1/2*(2*A*a^2*c*e^2-2*A*a
*b*c*d*e-2*A*a*c^2*d^2+A*b^2*c*d^2-B*a^2*b*e^2+4*B*a^2*c*d*e-B*a*b*c*d^2)/c
^2/(4*a*c-b^2)/a*x)/((x^4+1/c*b*x^2+a/c)*c)^(1/2)+1/4*(e*(A*c*e-B*b*e+2*B*c
*d)/c^2-1/c^2*(A*a*c*e^2-A*c^2*d^2-B*a*b*e^2+2*B*a*c*d*e)/a+1/a*(2*A*a^2*c*
e^2-2*A*a*b*c*d*e-2*A*a*c^2*d^2+A*b^2*c*d^2-B*a^2*b*e^2+4*B*a^2*c*d*e-B*a*b
*c*d^2)/c/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c
*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(1/c*B*e^2+1/a*(A*a*b
*c*e^2-4*A*a*c^2*d*e+A*b*c^2*d^2+2*B*a^2*c*e^2-B*a*b^2*e^2+2*B*a*b*c*d*e-2*
B*a*c^2*d^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-
2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1669 vs. 2(619) = 1238.

Time = 0.11 (sec) , antiderivative size = 1669, normalized size of antiderivative = 2.66

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/2*(sqrt(1/2)*(((2*B*a*b - A*b^2)*c^3*d^2 - 2*(B*a*b^2*c^2 - 2*A*a*b*c^3)
*d*e + (2*B*a*b^3*c - (6*B*a^2*b + A*a*b^2)*c^2)*e^2)*x^5 + ((2*B*a*b^2 - A
*b^3)*c^2*d^2 - 2*(B*a*b^3*c - 2*A*a*b^2*c^2)*d*e + (2*B*a*b^4 - (6*B*a^2*b
^2 + A*a*b^3)*c)*e^2)*x^3 + ((2*B*a^2*b - A*a*b^2)*c^2*d^2 - 2*(B*a^2*b^2*c
 - 2*A*a^2*b*c^2)*d*e + (2*B*a^2*b^3 - (6*B*a^3*b + A*a^2*b^2)*c)*e^2)*x -
(((2*B*a - A*b)*c^4*d^2 - 2*(B*a*b*c^3 - 2*A*a*c^4)*d*e + (2*B*a*b^2*c^2 -
(6*B*a^2 + A*a*b)*c^3)*e^2)*x^5 + ((2*B*a*b - A*b^2)*c^3*d^2 - 2*(B*a*b^2*c
^2 - 2*A*a*b*c^3)*d*e + (2*B*a*b^3*c - (6*B*a^2*b + A*a*b^2)*c^2)*e^2)*x^3
+ ((2*B*a^2 - A*a*b)*c^3*d^2 - 2*(B*a^2*b*c^2 - 2*A*a^2*c^3)*d*e + (2*B*a^2
*b^2*c - (6*B*a^3 + A*a^2*b)*c^2)*e^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*
sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c
*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2
 - 2*a*c)/(a*c)) + sqrt(1/2)*(((2*A*b*c^4 - (2*B*a*b - (A - B)*b^2)*c^3)*d^
2 + 2*(B*a*b^2*c^2 - (2*(A - B)*a*b + A*b^2)*c^3)*d*e - (2*B*a*b^3*c - 2*A*
a*b*c^3 - (6*B*a^2*b + (A - B)*a*b^2)*c^2)*e^2)*x^5 + ((2*A*b^2*c^3 - (2*B*
a*b^2 - (A - B)*b^3)*c^2)*d^2 + 2*(B*a*b^3*c - (2*(A - B)*a*b^2 + A*b^3)*c^
2)*d*e - (2*B*a*b^4 - 2*A*a*b^2*c^2 - (6*B*a^2*b^2 + (A - B)*a*b^3)*c)*e^2)
*x^3 + ((2*A*a*b*c^3 - (2*B*a^2*b - (A - B)*a*b^2)*c^2)*d^2 + 2*(B*a^2*b^2*
c - (2*(A - B)*a^2*b + A*a*b^2)*c^2)*d*e - (2*B*a^2*b^3 - 2*A*a^2*b*c^2 - (
6*B*a^3*b + (A - B)*a^2*b^2)*c)*e^2)*x + (((2*A*c^5 + (2*B*a - (A + B)*b)*c
^4)*d^2 - 2*(B*a*b*c^3 - (2*(A + B)*a - A*b)*c^4)*d*e + (2*B*a*b^2*c^2 + 2*
A*a*c^4 - (6*B*a^2 + (A + B)*a*b)*c^3)*e^2)*x^5 + ((2*A*b*c^4 + (2*B*a*b -
(A + B)*b^2)*c^3)*d^2 - 2*(B*a*b^2*c^2 - (2*(A + B)*a*b - A*b^2)*c^3)*d*e +
 (2*B*a*b^3*c + 2*A*a*b*c^3 - (6*B*a^2*b + (A + B)*a*b^2)*c^2)*e^2)*x^3 + (
(2*A*a*c^4 + (2*B*a^2 - (A + B)*a*b)*c^3)*d^2 - 2*(B*a^2*b*c^2 - (2*(A + B)
*a^2 - A*a*b)*c^3)*d*e + (2*B*a^2*b^2*c + 2*A*a^2*c^3 - (6*B*a^3 + (A + B)*
a^2*b)*c^2)*e^2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt(c)*sqrt((c*sqrt((b^2 - 4*
a*c)/c^2) - b)/c)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - 2*(
(B*a*b^2*c^2 - 4*B*a^2*c^3)*e^2*x^4 + (2*B*a^2 - A*a*b)*c^3*d^2 - 2*(B*a^2*
b*c^2 - 2*A*a^2*c^3)*d*e + (2*B*a^2*b^2*c - (6*B*a^3 + A*a^2*b)*c^2)*e^2 +
((B*a*b*c^3 - 2*A*a*c^4)*d^2 - 2*(B*a*b^2*c^2 - (2*B*a^2 + A*a*b)*c^3)*d*e
+ (2*B*a*b^3*c + 2*A*a^2*c^3 - (7*B*a^2*b + A*a*b^2)*c^2)*e^2)*x^2)*sqrt(c*
x^4 + b*x^2 + a))/((a*b^2*c^4 - 4*a^2*c^5)*x^5 + (a*b^3*c^3 - 4*a^2*b*c^4)*
x^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*x)
```

Sympy [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^2}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)^2}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.26 $\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 481

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = -\frac{x(aB(bd-2ae)-A(b^2d-2acd-abe)-(Ac(bd-2ae)-aB(2cd-be))x^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{(Ac(bd-2ae)-aB(2cd-be))x\sqrt{a+bx^2+cx^4}}{a\sqrt{c}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)} + \frac{(Ac(bd-2ae)-aB(2cd-be))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}c^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{(\sqrt{a}B-A\sqrt{c})(\sqrt{c}d-\sqrt{a}e)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})c^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -x*(a*B*(-2*a*e+b*d)-A*(-a*b*e-2*a*c*d+b^2*d)-(A*c*(-2*a*e+b*d)-a*B*(-b*e+2
*c*d))*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)-(A*c*(-2*a*e+b*d)-a*B*(-b*
e+2*c*d))*x*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/c^(1/2)/(a^(1/2)+x^2*c^(1/
2))+(A*c*(-2*a*e+b*d)-a*B*(-b*e+2*c*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a
^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x
^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/(-4*a*c+b^2)/(c*x^4+b*
x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/
2)/c^(1/2))^(1/2))*(B*a^(1/2)-A*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/
(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1692, 1211, 1117, 1209}

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)(Ac(bd-2ae)}{a^{3/4}c^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{(\sqrt{a}+\sqrt{c}x^2)(\sqrt{a}B-A\sqrt{c})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{c}d-\sqrt{a}e)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}c^{3/4}(b-2\sqrt{a}\sqrt{c})\sqrt{a+bx^2+cx^4}}$$
$$-\frac{x\sqrt{a+bx^2+cx^4}(Ac(bd-2ae)-aB(2cd-be))}{a\sqrt{c}(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{x(-A(-abe-2acd+b^2d)-(x^2(Ac(bd-2ae)-aB(2cd-be)))+aB(bd-2ae))}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(a*B*(b*d - 2*a*e) - A*(b^2*d - 2*a*c*d - a*b*e) - (A*c*(b*d - 2*a*e)
- a*B*(2*c*d - b*e))*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) - ((A
*c*(b*d - 2*a*e) - a*B*(2*c*d - b*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*Sqrt[c]
*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*c*(b*d - 2*a*e) - a*B*(2*c*d
- b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(a^(3/4)*c^(3/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B
- A*Sqrt[c])*(Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*c^(3/4
)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{x(aB(bd-2ae)-A(b^2d-2acd-abe)-(Ac(bd-2ae)-aB(2cd-be))x^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\int \frac{-a(bBd-2Acd+Abe-2aBe)+(Ac(bd-2ae)-aB(2cd-be))x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= -\frac{x(aB(bd-2ae)-A(b^2d-2acd-abe)-(Ac(bd-2ae)-aB(2cd-be))x^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} + \frac{\left(\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{c}} + \frac{(Ac(bd-2ae)-aB(2cd-be))\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\sqrt{c}\left(b^2-4ac\right)}$$

$$= -\frac{x(aB(bd-2ae)-A(b^2d-2acd-abe)-(Ac(bd-2ae)-aB(2cd-be))x^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{(Ac(bd-2ae)-aB(2cd-be))x\sqrt{a+bx^2+cx^4}}{a\sqrt{c}(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}$$
$$+\frac{(Ac(bd-2ae)-aB(2cd-be))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}c^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.66 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(aB(-2ae+2cdx^2+b(d-ex^2))+A(-b^2d+b(ae-cdx^2)+ \cdots}{\cdots}$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a*B*(-2*a*e + 2*c*d*x^2 + b*(d - e*
x^2)) + A*(-(b^2*d) + b*(a*e - c*d*x^2) + 2*a*c*(d + e*x^2))) + I*(-b + Sqr
t[b^2 - 4*a*c])*(A*c*(b*d - 2*a*e) + a*B*(-2*c*d + b*e))*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
] - I*(A*c*(-(b^2*d) + 4*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*Sqrt[b^2 - 4*a
*c]*e) + a*B*(b*(-b + Sqrt[b^2 - 4*a*c])*e + c*(-2*Sqrt[b^2 - 4*a*c]*d + 4*
a*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I
*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])])/(4*a*c*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.23

method	result
elliptic	$-\frac{2c\left(-\frac{(2Aace-Abcd-Babe+2Bacd)x^3}{2ca\left(4ac-b^2\right)}-\frac{\left(Aabe+2Acda-A\,b^2d-2eB\,a^2+Babd\right)x}{2ca\left(4ac-b^2\right)}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{eB}{c}+\frac{Acd-Bae}{ac}-\frac{Aabe+2Acda-A\,b^2d-2eB\,a^2+Babd}{a\left(4ac-b^2\right)}\right.}{}$
default	Expression too large to display

[In] int((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2*c*(-1/2/c*(2*A*a*c*e-A*b*c*d-B*a*b*e+2*B*a*c*d)/a/(4*a*c-b^2)*x^3-1/2/c*(A*a*b*e+2*A*a*c*d-A*b^2*d-2*B*a^2*e+B*a*b*d)/a/(4*a*c-b^2)*x)/((x^4+1/c*b*x^2+a/c)*c)^(1/2)+1/4*(e*B/c+(A*c*d-B*a*e)/a/c-(A*a*b*e+2*A*a*c*d-A*b^2*d-2*B*a^2*e+B*a*b*d)/a/(4*a*c-b^2))*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/2*(2*A*a*c*e-A*b*c*d-B*a*b*e+2*B*a*c*d)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1031 vs. 2(470) = 940.

Time = 0.10 (sec) , antiderivative size = 1031, normalized size of antiderivative = 2.14

$$\int \frac{(A+Bx^2)\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \text{Too large to display}$$

[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*(((2*B*a*b - A*b^2)*c^2*d - (B*a*b^2*c - 2*A*a*b*c^2)*e)*x^4 + (2*B*a^2*b - A*a*b^2)*c*d + ((2*B*a*b^2 - A*b^3)*c*d - (B*a*b^3 - 2*A*a*b^2*c)*e)*x^2 - (B*a^2*b^2 - 2*A*a^2*b*c)*e - (((2*B*a^2 - A*a*b)*c^2*d - (B*a^2*b*c - 2*A*a^2*c^2)*e)*x^4 + (2*B*a^3 - A*a^2*b)*c*d + ((2*B*a^2*b - A*a*b^2)*c*d - (B*a^2*b^2 - 2*A*a^2*b*c)*e)*x^2 - (B*a^3*b - 2*A*a^3*c)*e)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a

```
*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(((B*a*b^2*c -
 (2*(A - B)*a*b + A*b^2)*c^2)*d + (2*A*a*b*c^2 - (2*B*a^2*b - (A - B)*a*b^2
)*c)*e)*x^4 + ((B*a*b^3 - (2*(A - B)*a*b^2 + A*b^3)*c)*d - (2*B*a^2*b^2 - (
A - B)*a*b^3 - 2*A*a*b^2*c)*e)*x^2 + (B*a^2*b^2 - (2*(A - B)*a^2*b + A*a*b^
2)*c)*d - (2*B*a^3*b - (A - B)*a^2*b^2 - 2*A*a^2*b*c)*e + (((B*a^2*b*c - (2
*(A + B)*a^2 - A*a*b)*c^2)*d - (2*A*a^2*c^2 + (2*B*a^3 - (A + B)*a^2*b)*c)*
e)*x^4 + ((B*a^2*b^2 - (2*(A + B)*a^2*b - A*a*b^2)*c)*d - (2*B*a^3*b - (A +
 B)*a^2*b^2 + 2*A*a^2*b*c)*e)*x^2 + (B*a^3*b - (2*(A + B)*a^3 - A*a^2*b)*c)
*d - (2*B*a^4 - (A + B)*a^3*b + 2*A*a^3*c)*e)*sqrt((b^2 - 4*a*c)/a^2))*sqrt
(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_f(arcsin(sqrt(1/2)*x*s
qrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) +
 b^2 - 2*a*c)/(a*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*(((2*B*a^2 - A*a*b)*c^2*d
- (B*a^2*b*c - 2*A*a^2*c^2)*e)*x^3 - ((2*B*a^3 - A*a^2*b)*c*e - (2*A*a^2*c^
2 + (B*a^2*b - A*a*b^2)*c)*d)*x))/(a^3*b^2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4
*a^3*c^3)*x^4 + (a^2*b^3*c - 4*a^3*b*c^2)*x^2)
```

Sympy [F]

$$\int \frac{(A + Bx^2)\,(d + ex^2)}{(a + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{(A + Bx^2)\,(d + ex^2)}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((A + B*x**2)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{(A + Bx^2)\,(d + ex^2)}{(a + bx^2 + cx^4)^{3/2}}\,dx = \int \frac{(Bx^2 + A)(ex^2 + d)}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(Bx^2+A)(ex^2+d)}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{(B\,x^2+A)\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((A + B*x^2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.27 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 398

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}} - \frac{(Ab-2aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{(Ab-2aB)\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

```
[Out] x*(A*b^2-a*b*B-2*a*A*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1
/2)-(A*b-2*B*a)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2
*c^(1/2))+(A*b-2*B*a)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(B*a^(1/2)-A*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*
c^(1/2))^2)^(1/2)/a^(3/4)/c^(1/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/
2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1192, 1211, 1117, 1209}

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt[4]{c}(\sqrt{a}+\sqrt{c}x^2)(Ab-2aB)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{(\sqrt{a}+\sqrt{c}x^2)(\sqrt{a}B-A\sqrt{c})\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}(b-2\sqrt{a}\sqrt{c})\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt{c}x(Ab-2aB)\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)(\sqrt{a}+\sqrt{c}x^2)}+\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{a(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a
 + b*x^2 + c*x^4]) - ((A*b - 2*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(
b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) + ((A*b - 2*a*B)*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 -
 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*
Sqrt[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\int\frac{-a(bB-2Ac)+(Ab-2aB)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\left(b^2-4ac\right)}\\
&= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}+\frac{\left((Ab-2aB)\sqrt{c}\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b^2-4ac\right)}\\
&\quad-\frac{\left((Ab-2aB)\sqrt{c}-\sqrt{a}(bB-2Ac)\right)\int\frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\left(b^2-4ac\right)}\\
&= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{(Ab-2aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}\\
&\quad+\frac{(Ab-2aB)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}\\
&\quad+\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.91 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx =$$

$$-\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x(aB(b+2cx^2)-A(b^2-2ac+bcx^2))+i(Ab-2aB)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{2b}}{}$$

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/4*(4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a
*c + b*c*x^2)) + I*(A*b - 2*a*B)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a
*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - I*(-2*a*B*Sqrt[b^2 - 4*a*c] + A*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]))*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.28

method	result
elliptic	$-\frac{2c\left(\frac{(Ab-2Ba)x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2Aac-A\,b^2+abB\right)x}{2ac\left(4ac-b^2\right)}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{A}{a}-\frac{2Aac-A\,b^2+abB}{\left(4ac-b^2\right)a}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}}{}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$
default	$A\left(-\frac{2c\left(\frac{b\,x^3}{2a\left(4ac-b^2\right)}-\frac{\left(2ac-b^2\right)x}{2a\left(4ac-b^2\right)c}\right)}{\sqrt{\left(x^4+\frac{b\,x^2}{c}+\frac{a}{c}\right)c}}+\frac{\left(\frac{1}{a}-\frac{2ac-b^2}{a\left(4ac-b^2\right)}\right)\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}F\left(\frac{x\sqrt{2}\sqrt{-b+\sqrt{}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}\right.$

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(1/2*(A*b-2*B*a)/a/(4*a*c-b^2)*x^3-1/2*(2*A*a*c-A*b^2+B*a*b)/a/c/(4*a*
c-b^2)*x)/((x^4+1/c*b*x^2+a/c)*c)^(1/2)+1/4*(A/a-(2*A*a*c-A*b^2+B*a*b)/(4*a
*c-b^2)/a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*c*(A*b-2*B*a)/(4*a*c-b^2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\Big((2\,Bab-Ab^2)c^2x^4+(2\,Bab^2-Ab^3)cx^2+(2\,Ba^2b-Aab^2)c-((2\,Ba^2-A}{}$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*((2*B*a*b - A*b^2)*c^2*x^4 + (2*B*a*b^2 - A*b^3)*c*x^2 + (2*
B*a^2*b - A*a*b^2)*c - ((2*B*a^2 - A*a*b)*c^2*x^4 + (2*B*a^2*b - A*a*b^2)*c
*x^2 + (2*B*a^3 - A*a^2*b)*c)*sqrt((b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt
((b^2 - 4*a*c)/a^2) - b)/a)*elliptic_e(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2
 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a
*c)) - sqrt(1/2)*(B*a^2*b^2 + (B*a*b^2*c - (2*(A - B)*a*b + A*b^2)*c^2)*x^4
 + (B*a*b^3 - (2*(A - B)*a*b^2 + A*b^3)*c)*x^2 - (2*(A - B)*a^2*b + A*a*b^2
)*c + (B*a^3*b + (B*a^2*b*c - (2*(A + B)*a^2 - A*a*b)*c^2)*x^4 + (B*a^2*b^2
 - (2*(A + B)*a^2*b - A*a*b^2)*c)*x^2 - (2*(A + B)*a^3 - A*a^2*b)*c)*sqrt((
b^2 - 4*a*c)/a^2))*sqrt(a)*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)*elliptic
_f(arcsin(sqrt(1/2)*x*sqrt((a*sqrt((b^2 - 4*a*c)/a^2) - b)/a)), 1/2*(a*b*sq
rt((b^2 - 4*a*c)/a^2) + b^2 - 2*a*c)/(a*c)) - 2*((2*B*a^2 - A*a*b)*c^2*x^3
+ (2*A*a^2*c^2 + (B*a^2*b - A*a*b^2)*c)*x)*sqrt(c*x^4 + b*x^2 + a))/(a^3*b^
2*c - 4*a^4*c^2 + (a^2*b^2*c^2 - 4*a^3*c^3)*x^4 + (a^2*b^3*c - 4*a^3*b*c^2)
*x^2)
```

Sympy [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{A+Bx^2}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((A + B*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.28 $\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 867

$$
\begin{aligned}
&\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \\
&-\frac{x\left(abc(Bd-Ae)-(b^2-2ac)(Acd-Abe+aBe)+c(aB(2cd-be)-A(bcd-b^2e+2ace))x^2\right)}{a(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} \\
&+\frac{\sqrt{c}(aB(2cd-be)-A(bcd-b^2e+2ace))x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)(cd^2-bde+ae^2)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&-\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}(cd^2-bde+ae^2)^{3/2}} \\
&-\frac{\sqrt[4]{c}(aB(2cd-be)-A(bcd-b^2e+2ace))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} \\
&+\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{a+bx^2+cx^4}} \\
&+\frac{a^{3/4}e\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Bd-Ae)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{4\sqrt[4]{c}d(cd^2-ae^2)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

```
[Out] -1/2*e^(3/2)*(-A*e+B*d)*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)/d^(1/2)-x*(a*b*c*(-A*e+B*d
)-(-2*a*c+b^2)*(-A*b*e+A*c*d+B*a*e)+c*(a*B*(-b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*
c*d))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+(a*B*(-
b*e+2*c*d)-A*(2*a*c*e-b^2*e+b*c*d))*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(-4*a
```

```
*c+b^2)/(a*e^2-b*d*e+c*d^2)/(a^(1/2)+x^2*c^(1/2))-c^(1/4)*(a*B*(-b*e+2*c*d)
-A*(2*a*c*e-b^2*e+b*c*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^
2+a)^(1/2)+1/4*a^(3/4)*e*(-A*e+B*d)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1
/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1
/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^
(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4+b*x^2+a
)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(a*e^2-b*d*e+c*d^
2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(B*a^(1/2)-A*c^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-e*a^(1/2)
+d*c^(1/2))/(b-2*a^(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 1045, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1734, 1192, 1211, 1117, 1209, 1230, 1720}

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{a^{3/4}e(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\,\text{a}\right.}{4\sqrt[4]{c}d\left(cd^2-ae^2\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4}}$$
$$-\frac{e^{3/2}(Bd-Ae)\arctan\left(\frac{\sqrt{cd^2-bed+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+bx^2+a}}\right)}{2\sqrt{d}\left(cd^2-bed+ae^2\right)^{3/2}}$$
$$-\frac{\sqrt[4]{c}(aB(2cd-be)-A(-eb^2+cdb+2ace))\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$
$$-\frac{\sqrt[4]{c}\left(aBe-\sqrt{a}\sqrt{c}(Bd-Ae)+A(cd-be)\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\right.\right.}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$
$$-\frac{\sqrt[4]{c}e(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$
$$+\frac{\sqrt{c}(aB(2cd-be)-A(-eb^2+cdb+2ace))x\sqrt{cx^4+bx^2+a}}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)}$$
$$-\frac{x\left(c(aB(2cd-be)-A(-eb^2+cdb+2ace))x^2+abc(Bd-Ae)-\left(b^2-2ac\right)(Acd-Abe+aBe)\right)}{a\left(b^2-4ac\right)\left(cd^2-bed+ae^2\right)\sqrt{cx^4+bx^2+a}}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((x*(a*b*c*(B*d - A*e) - (b^2 - 2*a*c)*(A*c*d - A*b*e + a*B*e) + c*(a*B*(2
*c*d - b*e) - A*(b*c*d - b^2*e + 2*a*c*e))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 -
b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (Sqrt[c]*(a*B*(2*c*d - b*e) - A*
(b*c*d - b^2*e + 2*a*c*e))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(c*d
^2 - b*d*e + a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^(3/2)*(B*d - A*e)*ArcTan[
(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])
/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(3/2)) - (c^(1/4)*(a*B*(2*c*d - b*e) -
A*(b*c*d - b^2*e + 2*a*c*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
 b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sq
rt[a + b*x^2 + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a
]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(a*B*e - S
qrt[a]*Sqrt[c]*(B*d - A*e) + A*(c*d - b*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (a^(3/4)*e*((Sqrt[c]*d)
/Sqrt[a] + e)^2*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqr
t[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 +
c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1734

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

$$\text{integral} = \int \left(\frac{Acd - Abe + aBe + c(Bd - Ae)x^2}{(cd^2 - bde + ae^2)\left(a + bx^2 + cx^4\right)^{3/2}} + \frac{e(-Bd + Ae)}{(cd^2 - bde + ae^2)(d + ex^2)\sqrt{a + bx^2 + cx^4}} \right) dx$$

$$= \frac{\int \frac{Acd - Abe + aBe + c(Bd - Ae)x^2}{(a + bx^2 + cx^4)^{3/2}}\, dx}{cd^2 - bde + ae^2} - \frac{(e(Bd - Ae)) \int \frac{1}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\, dx}{cd^2 - bde + ae^2}$$

$$= -\frac{x\left(abc(Bd - Ae) - (b^2 - 2ac)(Acd - Abe + aBe) + c(aB(2cd - be) - A(bcd - b^2e + 2ace))x^2\right.}{a(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}}$$

$$- \frac{\int \frac{-ac(bBd - 2Acd + Abe - 2aBe) - c\left(aB(2cd - be) - A\left(bcd - b^2e + 2ace\right)\right)x^2}{\sqrt{a + bx^2 + cx^4}}\, dx}{a(b^2 - 4ac)(cd^2 - bde + ae^2)}$$

$$- \frac{\left(\sqrt{c}e(Bd - Ae)\right) \int \frac{1}{\sqrt{a + bx^2 + cx^4}}\, dx}{\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 - bde + ae^2)} + \frac{\left(\sqrt{a}e^2(Bd - Ae)\right) \int \frac{1 + \frac{\sqrt{c}x^2}{\sqrt{a}}}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\, dx}{\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 - bde + ae^2)}$$

$$= -\frac{x\left(abc(Bd - Ae) - (b^2 - 2ac)(Acd - Abe + aBe) + c(aB(2cd - be) - A(bcd - b^2e + 2ace))x^2\right.}{a(b^2 - 4ac)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}}$$

$$- \frac{e^{3/2}(Bd - Ae)\tan^{-1}\left(\frac{\sqrt{cd^2 - bde + ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{a + bx^2 + cx^4}}\right)}{2\sqrt{d}\left(cd^2 - bde + ae^2\right)^{3/2}}$$

$$- \frac{\sqrt[4]{c}e(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + bx^2 + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}}\, F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(2 - \frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}}$$

$$+ \frac{\sqrt[4]{a}e\left(\frac{\sqrt{c}d}{\sqrt{a}} + e\right)(Bd - Ae)\left(\sqrt{a} + \sqrt{c}x^2\right)\sqrt{\frac{a + bx^2 + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}}\, \Pi\left(-\frac{\left(\sqrt{c}d - \sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de}; 2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{4}\left(2 - \right.\right.}{4\sqrt[4]{c}d\left(\sqrt{c}d - \sqrt{a}e\right)(cd^2 - bde + ae^2)\sqrt{a + bx^2 + cx^4}}$$

$$- \frac{\left(\sqrt{c}\left(aBe - \sqrt{a}\sqrt{c}(Bd - Ae) + A(cd - be)\right)\right) \int \frac{1}{\sqrt{a + bx^2 + cx^4}}\, dx}{\sqrt{a}\left(b - 2\sqrt{a}\sqrt{c}\right)(cd^2 - bde + ae^2)}$$

$$- \frac{\left(\sqrt{c}(aB(2cd - be) - A(bcd - b^2e + 2ace))\right) \int \frac{1 - \frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a + bx^2 + cx^4}}\, dx}{\sqrt{a}(b^2 - 4ac)(cd^2 - bde + ae^2)}$$

$$= -\frac{x(abc(Bd-Ae)-(b^2-2ac)(Acd-Abe+aBe)+c(aB(2cd-be)-A(bcd-b^2e+2ace))x^2)}{a(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt{c}(aB(2cd-be)-A(bcd-b^2e+2ace))x\sqrt{a+bx^2+cx^4}}{a(b^2-4ac)(cd^2-bde+ae^2)(\sqrt{a}+\sqrt{c}x^2)}$$
$$-\frac{e^{3/2}(Bd-Ae)\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}(cd^2-bde+ae^2)^{3/2}}$$
$$-\frac{\sqrt[4]{c}(aB(2cd-be)-A(bcd-b^2e+2ace))(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\right.\right.}{a^{3/4}(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{c}e(Bd-Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\sqrt[4]{c}\left(aBe-\sqrt{a}\sqrt{c}(Bd-Ae)+A(cd-be)\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{2a^{3/4}(b-2\sqrt{a}\sqrt{c})(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$
$$+\frac{\sqrt[4]{a}e\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(Bd-Ae)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\Pi\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{}{\sqrt{c}}\right.\right.}{4\sqrt[4]{c}d(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.30 (sec) , antiderivative size = 1736, normalized size of antiderivative = 2.00

$$\int \frac{A+Bx^2}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{-4Ab^2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d^2x+4abBc\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d^2x+8aAc^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d^2x}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-4*A*b^2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x + 4*a*b*B*c*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*d^2*x + 8*a*A*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x
+ 4*A*b^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x - 4*a*b^2*B*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*d*e*x - 12*a*A*b*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x +
 8*a^2*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x - 4*A*b*c^2*Sqrt[c/(b + Sq
rt[b^2 - 4*a*c])]*d^2*x^3 + 8*a*B*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d^2*x
^3 + 4*A*b^2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*x^3 - 4*a*b*B*c*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*d*e*x^3 - 8*a*A*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*d*e*x^3 - I*(-b + Sqrt[b^2 - 4*a*c])*d*(a*B*(2*c*d - b*e) + A*(-(b*c*d) +
```

```
b^2*e - 2*a*c*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*d*(a*B*(b*(b - Sqrt[b^2 - 4*a*c])
*e + 2*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e)) + A*(-(b^3*e) + b*c*(-(Sqrt[b^2 - 4
*a*c]*d) + 4*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - 2*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (2*I)*a*b^2*B*d*e*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*
c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (8*I)*a^2*B*c*d*e*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (2*I)*a*A*b^2*e^2*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*
a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (8*I)*a^2*A*c*e^2*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 -
 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x],
(b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*(-b^2 + 4*a*c)*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*
x^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3240 vs. $2(832) = 1664$.

Time = 1.76 (sec) , antiderivative size = 3241, normalized size of antiderivative = 3.74

method	result	size
default	Expression too large to display	3241
elliptic	Expression too large to display	4597

```
[In] int((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B/e*(-2*c*(1/2/a*b/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/((x^4
+1/c*b*x^2+a/c)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
```

```
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-1/2*b*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/
2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+(A*e-B*d)/e*(-2*c*(1/2*(2*a*c*e-b^
2*e+b*c*d)/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)*x^3+1/2*(3*a*b*c*e-2*a*c^2*d-b
^3*e+b^2*c*d)/a/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)/c*x)/((x^4+1/c*b*x^2+a/c)*c
)^(1/2)-1/4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/
a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*b
*e+1/4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4
*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^
4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*c*d+3/
4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+
b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/(4*a*c-b^2)/(a*e^2-b*d*e+c*d^2)*b
*c*e-1/2*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(
-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/(4*a*c-b^2)/(a*e^2-b*d*e+c
*d^2)*c^2*d-1/4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*
x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(4*a*c-b^2)/(a*e^
2-b*d*e+c*d^2)*b^3*e+1/4*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b
*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^
(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))/a/(4*a*c-b
^2)/(a*e^2-b*d*e+c*d^2)*b^2*c*d-c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)
/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2
))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*a*e+1/2*c/(a*e^2-b*
d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x
^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1
/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a
/c)^(1/2))*b^2*e-1/2*c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(-b/a+1/a*
(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4
+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a
```

```
*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b*d+c^2/(a*e^2-b*d*e+c*d^2)/(4
*a*c-b^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*
(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c
*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*a*
e-1/2*c/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2
))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/
a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*El
lipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*
a*c+b^2)^(1/2))/a/c)^(1/2))*b^2*e+1/2*c^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)*2
^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2
)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))*b*d+1/(a*e^2-
b*d*e+c*d^2)*e^2/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2
/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2
)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\text{Timed out}$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}(ex^2+d)}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.29 $\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 1301

$$\int \frac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{x(abc(Ae(2cd-be)-B(cd^2-ae^2))+(b^2-2ac)\left(aBe(2cd-be)+A\right.}{}$$

$$+\frac{\sqrt{c}(aBd(-4c^2d^2-3b^2e^2+4ce(bd+2ae))+A(2b^3de^2+2bcd(cd^2-3ae^2)-4ace(-2cd^2+ae^2)+b^2(-4c}{2a\left(-b^2+4ac\right)d\left(cd^2+e(-bd+ae)\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

$$-\frac{e^3(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}$$

$$+\frac{e^{3/2}(Ae(7cd^2-e(4bd-ae))-Bd(5cd^2-e(2bd+ae)))\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(cd^2-bde+ae^2\right)^{5/2}}$$

$$-\frac{\sqrt[4]{c}(aBd(4c^2d^2+3b^2e^2-4ce(bd+2ae))-A(2b^3de^2+2bcd(cd^2-3ae^2)-4ace(-2cd^2+ae^2)+b^2(-4cd}{2a^{3/4}\left(b^2-4ac\right)d\left(cd^2+e(-bd+ae)\right)^2\sqrt{a+}}$$

$$+\frac{\sqrt[4]{c}\left(a\sqrt{c}e(Bd-2Ae)+\sqrt{a}(Bd-Ae)(cd-be)+A\sqrt{c}d(-cd+be)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)d\left(-\sqrt{c}d+\sqrt{a}e\right)\left(-cd^2+e(bd-ae)\right)\sqrt{a+bx^2+cx^4}}$$

$$-\frac{e\left(\sqrt{c}d+\sqrt{a}e\right)\left(Ae(7cd^2-e(4bd-ae))-Bd(5cd^2-e(2bd+ae))\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx^2+cx}}$$

```
[Out] 1/4*e^(3/2)*(A*e*(7*c*d^2-e*(-a*e+4*b*d))-B*d*(5*c*d^2-e*(a*e+2*b*d)))*arct
an(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(3/
2)/(a*e^2-b*d*e+c*d^2)^(5/2)+x*(a*b*c*(A*e*(-b*e+2*c*d)-B*(-a*e^2+c*d^2))+(
-2*a*c+b^2)*(a*B*e*(-b*e+2*c*d)+A*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d)))-c*(a*B
*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+A*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*e^2-b*c
```

```
*(-3*a*e^2+c*d^2)))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+b*x^2+
a)^(1/2)-1/2*e^3*(-A*e+B*d)*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)^2
/(e*x^2+d)+1/2*(a*B*d*(-4*c^2*d^2-3*b^2*e^2+4*c*e*(2*a*e+b*d))+A*(2*b^3*d*e
^2+2*b*c*d*(-3*a*e^2+c*d^2)-4*a*c*e*(a*e^2-2*c*d^2)+b^2*(a*e^3-4*c*d^2*e)))
*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(4*a*c-b^2)/d/(c*d^2+e*(a*e-b*d))^2/(a^(
1/2)+x^2*c^(1/2))-1/2*c^(1/4)*(a*B*d*(4*c^2*d^2+3*b^2*e^2-4*c*e*(2*a*e+b*d)
)-A*(2*b^3*d*e^2+2*b*c*d*(-3*a*e^2+c*d^2)-4*a*c*e*(a*e^2-2*c*d^2)+b^2*(a*e^
3-4*c*d^2*e)))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1
/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)
/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/d/(c*d^2+e*(a*e-b*d))^2/(c*x^4+b*x^2+a)^(1
/2)-1/8*e*(A*e*(7*c*d^2-e*(-a*e+4*b*d))-B*d*(5*c*d^2-e*(a*e+2*b*d)))*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^
(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/
2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(
1/4)/d^2/(a*e^2-b*d*e+c*d^2)^2/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*((-A*e+B*d)*(-b*e+c*d)*a^(1/2)+a*e*(-2*A*e+B*d)*c^(1/2)+A*d*
(b*e-c*d)*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/a^(3/4)/d/(-c*d^2+e*(-a*e+b*d))/(e*a^(1/2)-d*c^(1/2))/(b-2*a^
(1/2)*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 2112, normalized size of antiderivative = 1.62, number of steps used = 15, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules

used = {1734, 1192, 1211, 1117, 1209, 1237, 1728, 1722, 1720, 1230}

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+bx^2+cx^4)^{3/2}}\,dx = -\frac{(Bd-Ae)x\sqrt{cx^4+bx^2+a}e^3}{2d(cd^2-bed+ae^2)^2(ex^2+d)}$$

$$-\frac{\sqrt[4]{a}\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)e^2}{2d(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+a}}$$

$$+\frac{\sqrt{c}(Bd-Ae)x\sqrt{cx^4+bx^2+a}e^2}{2d(cd^2-bed+ae^2)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

$$-\frac{(Bd-Ae)(3cd^2-e(2bd-ae))\arctan\left(\frac{\sqrt{cd^2-bed+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+bx^2+a}}\right)e^{3/2}}{4d^{3/2}(cd^2-bed+ae^2)^{5/2}}$$

$$+\frac{(Ae(2cd-be)-B(cd^2-ae^2))\arctan\left(\frac{\sqrt{cd^2-bed+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{cx^4+bx^2+a}}\right)e^{3/2}}{2\sqrt{d}(cd^2-bed+ae^2)^{5/2}}$$

$$+\frac{\sqrt[4]{c}(Ae(2cd-be)-B(cd^2-ae^2))\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right.}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+a}}$$

$$-\frac{\sqrt[4]{c}(Bd-Ae)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)e}{2\sqrt[4]{a}d\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-bed+ae^2)\sqrt{cx^4+bx^2+a}}$$

$$+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)(Bd-Ae)(3cd^2-e(2bd-ae))\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\,\text{arcta}\right.}{8\sqrt[4]{a}\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+a}}$$

$$-\frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)^2(Ae(2cd-be)-B(cd^2-ae^2))\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{c}x^2+\sqrt{a})^2}}\,\text{EllipticPi}\left(-\frac{(\sqrt{c}d-\sqrt{a}e)^2}{4\sqrt{a}\sqrt{c}de},2\,\text{arct}\right.}{4\sqrt[4]{c}d(cd^2-ae^2)(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+a}}$$

$$-\frac{\sqrt[4]{c}(aB(2c^2d^2+b^2e^2-2ce(bd+ae))+A(-e^2b^3+2cdeb^2-c(cd^2-3ae^2)b-4ac^2de))\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{c}{(}}}{a^{3/4}(b^2-4ac)(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+a}}$$

$$-\frac{\sqrt[4]{c}\left(a^{3/2}B\sqrt{c}e^2+a(2Bcd-bBe-Ace)e+A(cd-be)^2-\sqrt{a}\sqrt{c}(Bcd^2-Ae(2cd-be))\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{}}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)(cd^2-bed+ae^2)^2\sqrt{cx^4+bx^2+}}$$

$$+\frac{\sqrt{c}(aB(2c^2d^2+b^2e^2-2ce(bd+ae))+A(-e^2b^3+2cdeb^2-c(cd^2-3ae^2)b-4ac^2de))x\sqrt{cx^4+bx^2+a}}{a(b^2-4ac)(cd^2-bed+ae^2)^2\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

$$+\frac{x(-c(aB(2c^2d^2+b^2e^2-2ce(bd+ae))+A(-e^2b^3+2cdeb^2-c(cd^2-3ae^2)b-4ac^2de))x^2+abc(Ae(2cd}{a(b^2-4ac)(cd^2-bed+ae^2)^2\sqrt{}}$$

```
[In] Int[(A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(a*b*c*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2)) + (b^2 - 2*a*c)*(a*B*e*(2
*c*d - b*e) + A*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e))) - c*(a*B*(2*c^2*d^
2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e - b^3*e^2 -
 b*c*(c*d^2 - 3*a*e^2)))*x^2))/(a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)^2*S
qrt[a + b*x^2 + c*x^4]) + (Sqrt[c]*e^2*(B*d - A*e)*x*Sqrt[a + b*x^2 + c*x^4
])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (Sqrt[c]*(a*B*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e -
b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)))*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*
c)*(c*d^2 - b*d*e + a*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)) - (e^3*(B*d - A*e)*x*
Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) - (e^(
3/2)*(B*d - A*e)*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*d^2 - b*d*e + a
*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3/2)*(c*d^2 - b*
d*e + a*e^2)^(5/2)) + (e^(3/2)*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2))*ArcT
an[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4]
)])/(2*Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^(5/2)) - (a^(1/4)*c^(1/4)*e^2*(B*d -
 A*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4
])/(2*d*(c*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(a*B*
(2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e)) + A*(2*b^2*c*d*e - 4*a*c^2*d*e -
b^3*e^2 - b*c*(c*d^2 - 3*a*e^2)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*e*(B*d - A*e)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d
 - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*e
*(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c
*d^2 - b*d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(a^(3/2)*B*Sqrt
[c]*e^2 + A*(c*d - b*e)^2 + a*e*(2*B*c*d - b*B*e - A*c*e) - Sqrt[a]*Sqrt[c]
*(B*c*d^2 - A*e*(2*c*d - b*e)))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*(c*d^2 - b*
d*e + a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) + (e*(Sqrt[c]*d + Sqrt[a]*e)*(B*d -
 A*e)*(3*c*d^2 - e*(2*b*d - a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^
2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e +
a*e^2)^2*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*e*((Sqrt[c]*d)/Sqrt[a] + e)^2*
(A*e*(2*c*d - b*e) - B*(c*d^2 - a*e^2))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[
a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[
a]*Sqrt[c]))/4])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*(c*d^2 - b*d*e + a*e^2)^2*Sqr
t[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
```

```
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1734

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2 + c*x^4], Px*(d + e
*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, b, c, d, e}, x]
&& PolyQ[Px, x^2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
```

```
& IntegerQ[p + 1/2] && IntegerQ[q]
```

Rubi steps

integral

$$= \int \left(\frac{aBe(2cd-be)+A(c^2d^2+b^2e^2-ce(2bd+ae))-c(Ae(2cd-be)-B(cd^2-ae^2))x^2}{(cd^2-bde+ae^2)^2(a+bx^2+cx^4)^{3/2}} + \frac{e(-Bd+Ae)}{(cd^2-bde+ae^2)(d+ex^2)^2\sqrt{a+bx^2+cx^4}} + \frac{e(Ae(2cd-be)-B(cd^2-ae^2))}{(cd^2-bde+ae^2)^2(d+ex^2)\sqrt{a+bx^2+cx^4}} \right) dx$$

$$= \frac{\int \frac{aBe(2cd-be)+A(c^2d^2+b^2e^2-ce(2bd+ae))-c(Ae(2cd-be)-B(cd^2-ae^2))x^2}{(a+bx^2+cx^4)^{3/2}}\,dx}{(cd^2-bde+ae^2)^2} - \frac{(e(Bd-Ae))\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx}{cd^2-bde+ae^2} + \frac{(e(Ae(2cd-be)-B(cd^2-ae^2)))\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{(cd^2-bde+ae^2)^2}$$

$$= \frac{x(abc(Ae(2cd-be)-B(cd^2-ae^2))+(b^2-2ac)(aBe(2cd-be)+A(c^2d^2+b^2e^2-ce(2bd+ae)}{a(b^2-4ac)(cd^2-bde+a} - \frac{e^3(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d(cd^2-bde+ae^2)^2(d+ex^2)} - \frac{\int \frac{ac(Ab^2e^2+2c(Acd^2+2aBde-aAe^2)-b(Bcd^2+2Acde+aBe^2))-c(aB(2c^2d^2+b^2e^2-2ce(bd+ae))+A(2b^2cde-4ac^2de-b^3e^2-bc}{\sqrt{a+bx^2+cx^4}}}{a(b^2-4ac)(cd^2-bde+ae^2)^2} + \frac{(e(Bd-Ae))\int \frac{-2cd^2+e(2bd-ae)+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{2d(cd^2-bde+ae^2)^2} + \frac{(\sqrt{c}e(Ae(2cd-be)-B(cd^2-ae^2)))\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)^2} - \frac{(\sqrt{a}e^2(Ae(2cd-be)-B(cd^2-ae^2)))\int \frac{1+\frac{\sqrt{c}x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{(\sqrt{c}d-\sqrt{a}e)(cd^2-bde+ae^2)^2}$$

$$= \frac{x(abc(Ae(2cd-be)-B(cd^2-ae^2))+(b^2-2ac)\left(aBe(2cd-be)+A(c^2d^2+b^2e^2-ce(2bd+ae))\right.}{a\left(b^2-4ac\right)\left(cd^2-bde+ae\right.}$$

$$-\frac{e^3(Bd-Ae)x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}$$

$$+\frac{e^{3/2}(Ae(2cd-be)-B(cd^2-ae^2))\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\left(cd^2-bde+ae^2\right)^{5/2}}$$

$$+\frac{\sqrt[4]{c}e(Ae(2cd-be)-B(cd^2-ae^2))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right.}{2\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx^2+cx^4}}$$

$$-\frac{\sqrt[4]{a}e\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)(Ae(2cd-be)-B(cd^2-ae^2))\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\right.}{4\sqrt[4]{c}d\left(\sqrt{c}d-\sqrt{a}e\right)\left(cd^2-bde+ae^2\right)^2\sqrt{a+bx^2+cx^4}}$$

$$+\frac{(Bd-Ae)\int\frac{\sqrt{a}c^{3/2}de^2+ce(-2cd^2+e(2bd-ae))+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\sqrt{c}e\right)\right)x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}dx}{2cd\left(cd^2-bde+ae^2\right)^2}$$

$$-\frac{\left(\sqrt{a}\sqrt{c}e^2(Bd-Ae)\right)\int\frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}dx}{2d\left(cd^2-bde+ae^2\right)^2}$$

$$-\frac{\left(\sqrt{c}\left(a^{3/2}B\sqrt{c}e^2+A(cd-be)^2+ae(2Bcd-bBe-Ace)-\sqrt{a}\sqrt{c}(Bcd^2-Ae(2cd-be))\right)\right)\int\frac{}{\sqrt{a}}}{\sqrt{a}\left(b-2\sqrt{a}\sqrt{c}\right)\left(cd^2-bde+ae^2\right)^2}$$

$$-\frac{\left(\sqrt{c}\left(aB(2c^2d^2+b^2e^2-2ce(bd+ae))+A(2b^2cde-4ac^2de-b^3e^2-bc(cd^2-3ae^2))\right)\right)\int\frac{1-\frac{\sqrt{c}x}{\sqrt{c}}}{\sqrt{a+bx^2-}}}{\sqrt{a}\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2}$$

= Too large to display

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 15.22 (sec) , antiderivative size = 1116, normalized size of antiderivative = 0.86

$$\int\frac{A+Bx^2}{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d\left(-a\left(b^2-4ac\right)e^3(Bd-Ae)x\left(a+bx^2+cx^4\right)+2dx(d+e\right.}{}$$

```
[In] Integrate[(A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*(-(a*(b^2 - 4*a*c)*e^3*(B*d - A*e)*x*(
a + b*x^2 + c*x^4)) + 2*d*x*(d + e*x^2)*(a*B*(-(b^3*e^2) + b^2*c*e*(2*d - e
*x^2) + b*c*(3*a*e^2 - c*d*(d - 2*e*x^2)) - 2*c^2*(c*d^2*x^2 + a*e*(2*d - e
```

```
*x^2))) + A*(b^4*e^2 + b^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2
*e*x^2)) + b^2*c*(-4*a*e^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*
(-2*d + e*x^2))))) - I*Sqrt[2]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)
*((-b + Sqrt[b^2 - 4*a*c])*d*(a*B*d*(-4*c^2*d^2 - 3*b^2*e^2 + 4*c*e*(b*d +
2*a*e)) + A*(2*b^3*d*e^2 + 2*b*c*d*(c*d^2 - 3*a*e^2) - 4*a*c*e*(-2*c*d^2 +
a*e^2) + b^2*(-4*c*d^2*e + a*e^3)))*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b +
 Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] -
 d*(a*B*d*(3*b^2*(b - Sqrt[b^2 - 4*a*c])*e^2 - 4*c^2*d*(Sqrt[b^2 - 4*a*c]*d
 - 6*a*e) + 2*c*(-3*b + 2*Sqrt[b^2 - 4*a*c])*e*(b*d + 2*a*e)) + A*(-2*b^4*d
*e^2 + b^2*(-2*c^2*d^3 + a*Sqrt[b^2 - 4*a*c]*e^3 - 4*c*d*e*(Sqrt[b^2 - 4*a*
c]*d - 3*a*e)) - 4*a*c*(-2*c^2*d^3 + a*Sqrt[b^2 - 4*a*c]*e^3 - 2*c*d*e*(Sqr
t[b^2 - 4*a*c]*d - 2*a*e)) + b^3*e*(4*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*
e)) + 2*b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + a*e^2*(-3*Sqrt[b^2 - 4*a
*c]*d + 2*a*e))))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - 2*a*(-b^2 + 4*a*c
)*e*(A*e*(7*c*d^2 + e*(-4*b*d + a*e)) + B*(-5*c*d^3 + d*e*(2*b*d + a*e)))*E
llipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])
)/(8*a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(c*d^3 + d*e*(-(b*d) +
 a*e))^2*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 8275 vs. $2(1252) = 2504$.

Time = 3.61 (sec) , antiderivative size = 8276, normalized size of antiderivative = 6.36

method	result	size
default	Expression too large to display	8276
elliptic	Expression too large to display	9725

```
[In] int((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2 (a+bx^2+cx^4)^{3/2}} \, dx = \text{Timed out}$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+bx^2+cx^4)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}\,(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Giac [F]

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{Bx^2+A}{(cx^4+bx^2+a)^{\frac{3}{2}}\,(ex^2+d)^2}\,dx$$

```
[In] integrate((B*x^2+A)/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2}{(d+ex^2)^2\,(a+bx^2+cx^4)^{3/2}}\,dx = \int \frac{B\,x^2+A}{(e\,x^2+d)^2\,(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int((A + B*x^2)/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.30 $\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 273

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}$$

```
[Out] -1/2*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/
2))*(-e*a^(1/2)+d*c^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/
e/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a
^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)
/c^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used

$= \{1720\}$

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[
d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e +
 a*e^2]) + ((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e
)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*S
qrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$
$$+\frac{\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)^2}{4\sqrt{a}\sqrt{c}de};2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{c}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(-\sqrt{c}d\cdots}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+c\cdots}}$$

[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c]*d*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-(Sqrt[c]*d) + Sqrt[a]*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])

Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{c}\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}+\frac{(e\sqrt{a}-d\sqrt{c})\sqrt{2}\cdots}{\cdots}$
elliptic	$\frac{\sqrt{(cx^4+bx^2+a)c}\sqrt{(cx^4+bx^2+a)a}\,(\sqrt{a}+x^2\sqrt{c})\left(\frac{a\sqrt{2}\sqrt{1+\frac{bx^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\Pi\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},-\cdots\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\sqrt{acx^4+abx^2+a^2}}\cdots\right.}{\cdots}$

[In] int((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*c^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-

```
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/e*(e*a^(1/2)-d*c^(1/2))/d*2^(1/2
)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)
^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(
-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((a**(1/2)+x**2*c**(1/2))/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((sqrt(a) + sqrt(c)*x**2)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="maxima")
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Giac [F]

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a}+\sqrt{c}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{\sqrt{a}+\sqrt{c}\,x^2}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((a^(1/2) + c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a^(1/2) + c^(1/2)*x^2)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.31 $\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 271

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{\frac{c}{a}}d-e\right)\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}} + \frac{\left(\sqrt{\frac{c}{a}}d+e\right)\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(1+\sqrt{\frac{c}{a}}x^2\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4\sqrt[4]{\frac{c}{a}}de\sqrt{a+bx^2+cx^4}}$$

```
[Out] -1/2*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/
2))*(-e+d*(c/a)^(1/2))/d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/4*(cos(2
*arctan((c/a)^(1/4)*x))^2)^(1/2)/cos(2*arctan((c/a)^(1/4)*x))*EllipticPi(si
n(2*arctan((c/a)^(1/4)*x)),-1/4*(-e+d*(c/a)^(1/2))^2/d/e/(c/a)^(1/2),1/2*(2
-b*(c/a)^(1/2)/c)^(1/2))*(e+d*(c/a)^(1/2))*(1+x^2*(c/a)^(1/2))*((c*x^4+b*x^
2+a)/a/(1+x^2*(c/a)^(1/2))^2)^(1/2)/(c/a)^(1/4)/d/e/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used

$= \{1720\}$

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \frac{\left(x^2\sqrt{\frac{c}{a}}+1\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(x^2\sqrt{\frac{c}{a}}+1\right)^2}}\left(d\sqrt{\frac{c}{a}}+e\right)\text{EllipticPi}\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de},2\arctan\left(\sqrt[4]{\frac{c}{a}}x\right),\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4de\sqrt[4]{\frac{c}{a}}\sqrt{a+bx^2+cx^4}}$$
$$-\frac{\left(d\sqrt{\frac{c}{a}}-e\right)\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

```
[In] Int[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((Sqrt[c/a]*d - e)*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqr
t[e]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2
]) + ((Sqrt[c/a]*d + e)*(1 + Sqrt[c/a]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1
+ Sqrt[c/a]*x^2)^2)]*EllipticPi[-1/4*(Sqrt[c/a]*d - e)^2/(Sqrt[c/a]*d*e), 2
*ArcTan[(c/a)^(1/4)*x], (2 - (b*Sqrt[c/a])/c)/4])/(4*(c/a)^(1/4)*d*e*Sqrt[a
 + b*x^2 + c*x^4])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

integral
$$= -\frac{\left(\sqrt{\frac{c}{a}}d-e\right)\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$
$$+\frac{\left(\sqrt{\frac{c}{a}}d+e\right)\left(1+\sqrt{\frac{c}{a}}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{a\left(1+\sqrt{\frac{c}{a}}x^2\right)^2}}\Pi\left(-\frac{\left(\sqrt{\frac{c}{a}}d-e\right)^2}{4\sqrt{\frac{c}{a}}de};2\tan^{-1}\left(\sqrt[4]{\frac{c}{a}}x\right)|\frac{1}{4}\left(2-\frac{b\sqrt{\frac{c}{a}}}{c}\right)\right)}{4\sqrt[4]{\frac{c}{a}}de\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.15

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = -\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\sqrt{\frac{c}{a}}d\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(-\sqrt{\frac{c}{a}}d\right.\right.}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+cx^4}}$$

[In] Integrate[(1 + Sqrt[c/a]*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]

[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(Sqrt[c/a]*d*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-(Sqrt[c/a]*d) + e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4])

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.36

method	result
default	$\frac{\sqrt{\frac{c}{a}}\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac+b^2}\right)}{ac}}}{2}\right)}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{cx^4+bx^2+a}}+\frac{\left(e-d\sqrt{\frac{c}{a}}\right)\sqrt{2}\sqrt{1}}{}$
elliptic	$\frac{\sqrt{\frac{\left(cx^4+bx^2+a\right)c}{a}}\,a\left(1+x^2\sqrt{\frac{c}{a}}\right)\left(\frac{c\sqrt{2}\sqrt{4+\frac{2bx^2}{a}-\frac{2x^2\sqrt{-4ac+b^2}}{a}}\sqrt{4+\frac{2bx^2}{a}+\frac{2x^2\sqrt{-4ac+b^2}}{a}}\,F\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2},\frac{\sqrt{-4+\frac{2b\left(b+\sqrt{-4ac}\right.}{ac}}}{2}\right.}{4ae\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\sqrt{\frac{c^2x^4}{a}+\frac{bcx^2}{a}+c}}\right.}{}$

[In] int((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(c/a)^(1/2)/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/

```
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/e*(e-d*(c/a)^(1/2))/d*2^(1/2
)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)
^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x
^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(
-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \text{Timed out}$$

```
[In] integrate((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

```
[In] integrate((1+x**2*(c/a)**(1/2))/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((x**2*sqrt(c/a) + 1)/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Maxima [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Giac [F]

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

```
[In] integrate((1+x^2*(c/a)^(1/2))/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm=
"giac")
[Out] integrate((x^2*sqrt(c/a) + 1)/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{\frac{c}{a}}x^2}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{x^2\sqrt{\frac{c}{a}}+1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

```
[In] int((x^2*(c/a)^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((x^2*(c/a)^(1/2) + 1)/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.32 $\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 106

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{2525(2+x^2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

```
[Out] -2525/28*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2)
,2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+631/4
*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x
^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1718, 1113, 1470, 553}

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}} - \frac{2525(x^2+2)\,\text{EllipticPi}\left(\frac{2}{7},\arctan(x),\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

```
[In] Int[(946 + 315*x^2)/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt
[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2
])/(14*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1718

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Sqrt[b^2 - 4*a*c]}, Dist[(2*a*B - A
*(b + q))/(2*a*e - d*(b + q)), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[(B*d - A*e)/(2*a*e - d*(b + q)), Int[(2*a + (b + q)*x^2)/((d + e*x^2)*Sqrt
[a + b*x^2 + c*x^4]), x], x] /; RationalQ[q]] /; FreeQ[{a, b, c, d, e, A, B
}, x] && GtQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*A^2
- b*A*B + a*B^2, 0]
```

Rubi steps

$$\text{integral} = \frac{631}{2}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{2525}{8}\int \frac{4+4x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(2525\sqrt{\frac{1}{2}+\frac{x^2}{4}}\sqrt{4+4x^2}\right)\int \frac{\sqrt{4+4x^2}}{\sqrt{\frac{1}{2}+\frac{x^2}{4}}(7+5x^2)}\,dx}{8\sqrt{2+3x^2+x^4}}$$

$$= \frac{631(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{2525(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(441\,\mathrm{EllipticF}\left(i\,\mathrm{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+505\,\mathrm{EllipticPi}\left(\frac{10}{7},i\,\mathrm{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{7\sqrt{2+3x^2+x^4}}$$

```
[In] Integrate[(946 + 315*x^2)/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
[Out] ((-1/7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(441*EllipticF[I*ArcSinh[x/Sqrt[2]],
2] + 505*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{63i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}} - \frac{505i\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$	93
elliptic	$-\frac{63i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}} - \frac{505i\sqrt{2}\sqrt{1+\frac{x^2}{2}}\sqrt{x^2+1}\,\Pi\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^4+3x^2+2}}$	93

```
[In] int((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -63/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-505/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Fricas [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{315\,x^2+946}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(x^4 + 3*x^2 + 2)*(315*x^2 + 946)/(5*x^6 + 22*x^4 + 31*x^2 + 1
4), x)
```

Sympy [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{315x^2+946}{\sqrt{(x^2+1)\,(x^2+2)}\,(5x^2+7)}\,dx$$

```
[In] integrate((315*x**2+946)/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((315*x**2 + 946)/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

Maxima [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{315\,x^2+946}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima"
)
[Out] integrate((315*x^2 + 946)/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Giac [F]

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx = \int \frac{315\,x^2+946}{\sqrt{x^4+3\,x^2+2}(5\,x^2+7)}\,dx$$

```
[In] integrate((315*x^2+946)/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((315*x^2 + 946)/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{946+315x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx=\int \frac{315\,x^2+946}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

```
[In] int((315*x^2 + 946)/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)),x)
[Out] int((315*x^2 + 946)/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.33 $\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4} dx$

Optimal result

Integrand size = 31, antiderivative size = 218

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4} dx = \frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b-\sqrt{b^2-4ac}} + \frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\operatorname{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b+\sqrt{b^2-4ac}}$$

```
[Out] x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/
d)*(B+(2*A*c-B*b)/(-4*a*c+b^2)^(1/2))/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2)
)+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^
2/d)*(B+(-2*A*c+B*b)/(-4*a*c+b^2)^(1/2))/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1
/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used

$= \{1706, 441, 440\}$

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b-\sqrt{b^2-4ac}} + \frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\text{AppellF1}\left(\frac{1}{2},1,-q,\frac{3}{2},-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}+b}$$

```
[In] Int[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q,
 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/((b - Sqrt[b^2 - 4
*a*c])*(1 + (e*x^2)/d)^q) + ((B + (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*x*(d + e
*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*
x^2)/d)])/((b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule steps

$$\text{integral} = \int \left(\frac{\left(B+\frac{-bB+2Ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(B-\frac{-bB+2Ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx$$

$$= \left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right) \int \frac{(d + ex^2)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2} \, dx + \left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right) \int \frac{(d + ex^2)^q}{b + \sqrt{b^2 - 4ac} + 2cx^2} \, dx$$

$$= \left(\left(B - \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right) (d + ex^2)^q \left(1 + \frac{ex^2}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^2}{d}\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2} \, dx + \left(\left(B + \frac{bB - 2Ac}{\sqrt{b^2 - 4ac}}\right) (d + ex^2)^q \left(1 + \frac{ex^2}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^2}{d}\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^2} \, dx$$

$$= \frac{\left(B - \frac{bB-2Ac}{\sqrt{b^2-4ac}}\right) x(d + ex^2)^q \left(1 + \frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2}; 1, -q; \frac{3}{2}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{ex^2}{d}\right)}{b - \sqrt{b^2 - 4ac}} + \frac{\left(B + \frac{bB-2Ac}{\sqrt{b^2-4ac}}\right) x(d + ex^2)^q \left(1 + \frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2}; 1, -q; \frac{3}{2}; -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, -\frac{ex^2}{d}\right)}{b + \sqrt{b^2 - 4ac}}$$

Mathematica [F]

$$\int \frac{(A + Bx^2)(d + ex^2)^q}{a + bx^2 + cx^4} \, dx = \int \frac{(A + Bx^2)(d + ex^2)^q}{a + bx^2 + cx^4} \, dx$$

```
[In] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]

$$\int \frac{(B\,x^2 + A)(e\,x^2 + d)^q}{c\,x^4 + b\,x^2 + a} dx$$

```
[In] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \text{Timed out}$$

```
[In] integrate((B*x**2+A)*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(Bx^2+A)(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

```
[In] integrate((B*x^2+A)*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*(e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \frac{(B\,x^2+A)\,\left(e\,x^2+d\right)^q}{c\,x^4+b\,x^2+a}\,dx$$

```
[In] int(((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int(((A + B*x^2)*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.34 $\int \frac{x(1+2x^2)}{\sqrt{1+x^2}(1+x^2+x^4)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = -\frac{1}{2}\arctan\left(\sqrt{3}-2\sqrt{1+x^2}\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2\sqrt{1+x^2}\right)+\frac{1}{4}\sqrt{3}\log\left(2+x^2-\sqrt{3}\sqrt{1+x^2}\right)-\frac{1}{4}\sqrt{3}\log\left(2+x^2+\sqrt{3}\sqrt{1+x^2}\right)$$

```
[Out] 1/2*arctan(-3^(1/2)+2*(x^2+1)^(1/2))+1/2*arctan(3^(1/2)+2*(x^2+1)^(1/2))+1/
4*ln(2+x^2-3^(1/2)*(x^2+1)^(1/2))*3^(1/2)-1/4*ln(2+x^2+3^(1/2)*(x^2+1)^(1/2
))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1699, 840, 1183, 648, 632, 210, 642}

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = -\frac{1}{2}\arctan\left(\sqrt{3}-2\sqrt{x^2+1}\right)+\frac{1}{2}\arctan\left(2\sqrt{x^2+1}+\sqrt{3}\right)+\frac{1}{4}\sqrt{3}\log\left(x^2-\sqrt{3}\sqrt{x^2+1}+2\right)-\frac{1}{4}\sqrt{3}\log\left(x^2+\sqrt{3}\sqrt{x^2+1}+2\right)$$

```
[In] Int[(x*(1 + 2*x^2))/(Sqrt[1 + x^2]*(1 + x^2 + x^4)),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - 2*Sqrt[1 + x^2]] + ArcTan[Sqrt[3] + 2*Sqrt[1 + x^2]]/
2 + (Sqrt[3]*Log[2 + x^2 - Sqrt[3]*Sqrt[1 + x^2]])/4 - (Sqrt[3]*Log[2 + x^2
 + Sqrt[3]*Sqrt[1 + x^2]])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1699

```
Int[(Px_)*(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(Px /. x -> Sqrt[x])*(d + e*x)
^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
&& PolyQ[Px, x^2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1+2x}{\sqrt{1+x}\,(1+x+x^2)}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{-1+2x^2}{1-x^2+x^4}\,dx, x, \sqrt{1+x^2}\right)\\
&= \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+3x}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt{1+x^2}\right)}{2\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt{3}-3x}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt{1+x^2}\right)}{2\sqrt{3}}\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt{1+x^2}\right)\\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt{1+x^2}\right)\\
&\quad + \frac{1}{4}\sqrt{3}\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, \sqrt{1+x^2}\right)\\
&\quad - \frac{1}{4}\sqrt{3}\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, \sqrt{1+x^2}\right)\\
&= \frac{1}{4}\sqrt{3}\log\left(2+x^2-\sqrt{3}\sqrt{1+x^2}\right) - \frac{1}{4}\sqrt{3}\log\left(2+x^2+\sqrt{3}\sqrt{1+x^2}\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2\sqrt{1+x^2}\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2\sqrt{1+x^2}\right)\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2\sqrt{1+x^2}\right) + \frac{1}{2}\tan^{-1}\left(\sqrt{3}+2\sqrt{1+x^2}\right)\\
&\quad + \frac{1}{4}\sqrt{3}\log\left(2+x^2-\sqrt{3}\sqrt{1+x^2}\right) - \frac{1}{4}\sqrt{3}\log\left(2+x^2+\sqrt{3}\sqrt{1+x^2}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \frac{1}{2}\left(1-i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt{1+x^2}\right) + \frac{1}{2}\left(1+i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt{1+x^2}\right)$$

```
[In] Integrate[(x*(1 + 2*x^2))/(Sqrt[1 + x^2]*(1 + x^2 + x^4)),x]
```

```
[Out] ((1 - I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*Sqrt[1 + x^2])/2])/2 + ((1 + I*Sqr
t[3])*ArcTan[((1 + I*Sqrt[3])*Sqrt[1 + x^2])/2])/2
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{\arctan\left(-\sqrt{3}+2\sqrt{x^2+1}\right)}{2}+\frac{\arctan\left(\sqrt{3}+2\sqrt{x^2+1}\right)}{2}+\frac{\ln\left(2+x^2-\sqrt{3}\,\sqrt{x^2+1}\right)\sqrt{3}}{4}-\frac{\ln\left(2+x^2+\sqrt{3}\,\sqrt{x^2+1}\right)\sqrt{3}}{4}$
default	$-\frac{\sqrt{2}\,\sqrt{\frac{2(x-1)^2}{(-x-1)^2}+2}\left(\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{\frac{2(x-1)^2}{(-x-1)^2}+2}\,\sqrt{3}}{2}\right)+\arctan\left(\frac{\sqrt{\frac{2(x-1)^2}{(-x-1)^2}+2}\,(x-1)}{\left(\frac{(x-1)^2}{(-x-1)^2}+1\right)(-x-1)}\right)\right)}{4\sqrt{\frac{\frac{(x-1)^2}{(-x-1)^2}+1}{\left(\frac{x-1}{-x-1}+1\right)^2}}\left(\frac{x-1}{-x-1}+1\right)}-\frac{\sqrt{2}\,\sqrt{\frac{2(x+1)^2}{(1-x)^2}+2}\left(\sqrt{3}\,\text{arcta}\right.}{}$
trager	$-4\ln\left(\frac{-16\,\text{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^5x^2+16\,\text{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^3x^2+12\,\text{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^3-}{\left(4\,\text{RootOf}\left(16\_Z^4-4\_Z^2+1\right)^2x-x+1\right)\left(4\,\text{RootOf}\left(16\_Z\right.\right.}\right.$

```
[In] int(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(-3^(1/2)+2*(x^2+1)^(1/2))+1/2*arctan(3^(1/2)+2*(x^2+1)^(1/2))+1/
4*ln(2+x^2-3^(1/2)*(x^2+1)^(1/2))*3^(1/2)-1/4*ln(2+x^2+3^(1/2)*(x^2+1)^(1/2
))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(80) = 160.

Time = 0.33 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.70

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \frac{1}{4}\,\sqrt{2}\sqrt{-\sqrt{-3}+1}\log\left(4\,x^2+\sqrt{2}(\sqrt{-3}x+x)\sqrt{-\sqrt{-3}+1} - \sqrt{x^2+1}\left(\sqrt{2}(\sqrt{-3}+1)\sqrt{-\sqrt{-3}+1}+4\,x\right)+4\right) - \frac{1}{4}\,\sqrt{2}\sqrt{-\sqrt{-3}+1}\log\left(4\,x^2 - \sqrt{2}(\sqrt{-3}x+x)\sqrt{-\sqrt{-3}+1} + \sqrt{x^2+1}\left(\sqrt{2}(\sqrt{-3}+1)\sqrt{-\sqrt{-3}+1}-4\,x\right)+4\right) + \frac{1}{4}\,\sqrt{2}\sqrt{\sqrt{-3}+1}\log\left(4\,x^2-4\,\sqrt{x^2+1}x + \left(\sqrt{2}\sqrt{x^2+1}(\sqrt{-3}-1)-\sqrt{2}(\sqrt{-3}x-x)\right)\sqrt{\sqrt{-3}+1} + 4\right) - \frac{1}{4}\,\sqrt{2}\sqrt{\sqrt{-3}+1}\log\left(4\,x^2-4\,\sqrt{x^2+1}x - \left(\sqrt{2}\sqrt{x^2+1}(\sqrt{-3}-1)-\sqrt{2}(\sqrt{-3}x-x)\right)\sqrt{\sqrt{-3}+1} + 4\right)$$

```
[In] integrate(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt(-sqrt(-3) + 1)*log(4*x^2 + sqrt(2)*(sqrt(-3)*x + x)*sqrt(-
sqrt(-3) + 1) - sqrt(x^2 + 1)*(sqrt(2)*(sqrt(-3) + 1)*sqrt(-sqrt(-3) + 1) +
 4*x) + 4) - 1/4*sqrt(2)*sqrt(-sqrt(-3) + 1)*log(4*x^2 - sqrt(2)*(sqrt(-3)*
x + x)*sqrt(-sqrt(-3) + 1) + sqrt(x^2 + 1)*(sqrt(2)*(sqrt(-3) + 1)*sqrt(-sq
rt(-3) + 1) - 4*x) + 4) + 1/4*sqrt(2)*sqrt(sqrt(-3) + 1)*log(4*x^2 - 4*sqrt
(x^2 + 1)*x + (sqrt(2)*sqrt(x^2 + 1)*(sqrt(-3) - 1) - sqrt(2)*(sqrt(-3)*x -
 x))*sqrt(sqrt(-3) + 1) + 4) - 1/4*sqrt(2)*sqrt(sqrt(-3) + 1)*log(4*x^2 - 4
*sqrt(x^2 + 1)*x - (sqrt(2)*sqrt(x^2 + 1)*(sqrt(-3) - 1) - sqrt(2)*(sqrt(-3
)*x - x))*sqrt(sqrt(-3) + 1) + 4)
```

Sympy [F]

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \int \frac{x(2x^2+1)}{\sqrt{x^2+1}\,(x^2-x+1)\,(x^2+x+1)}\,dx$$

```
[In] integrate(x*(2*x**2+1)/(x**4+x**2+1)/(x**2+1)**(1/2),x)
[Out] Integral(x*(2*x**2 + 1)/(sqrt(x**2 + 1)*(x**2 - x + 1)*(x**2 + x + 1)), x)
```

Maxima [F]

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = \int \frac{(2\,x^2+1)x}{(x^4+x^2+1)\sqrt{x^2+1}}\,dx$$

```
[In] integrate(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)*x/((x^4 + x^2 + 1)*sqrt(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx = &-\frac{1}{4}\sqrt{3}\log\left(x^2+\sqrt{3}\sqrt{x^2+1}+2\right)\\ &+\frac{1}{4}\sqrt{3}\log\left(x^2-\sqrt{3}\sqrt{x^2+1}+2\right)\\ &+\frac{1}{2}\arctan\left(\sqrt{3}+2\sqrt{x^2+1}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2\sqrt{x^2+1}\right)\end{aligned}$$

```
[In] integrate(x*(2*x^2+1)/(x^4+x^2+1)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(3)*log(x^2 + sqrt(3)*sqrt(x^2 + 1) + 2) + 1/4*sqrt(3)*log(x^2 - s
qrt(3)*sqrt(x^2 + 1) + 2) + 1/2*arctan(sqrt(3) + 2*sqrt(x^2 + 1)) + 1/2*arc
tan(-sqrt(3) + 2*sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 8.63 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.75

$$\int \frac{x(1+2x^2)}{\sqrt{1+x^2}\,(1+x^2+x^4)}\,dx$$
$$=\frac{\left(\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(\frac{x}{2}+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}+1+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\left(4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+1+\sqrt{3}\,1\mathrm{i}\right)}$$
$$+\frac{\left(\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}+\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\left(4\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-1+\sqrt{3}\,1\mathrm{i}\right)}$$
$$+\frac{\left(\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(\frac{x}{2}+\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}+1-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\left(4\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3-1+\sqrt{3}\,1\mathrm{i}\right)}$$
$$+\frac{\left(\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\ln\left(1+\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\sqrt{x^2+1}-\frac{x}{2}-\frac{\sqrt{3}\,x\,1\mathrm{i}}{2}\right)\right)\left(2\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{\sqrt{\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2+1}\left(4\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^3+1+\sqrt{3}\,1\mathrm{i}\right)}$$

```
[In] int((x*(2*x^2 + 1))/((x^2 + 1)^(1/2)*(x^2 + x^4 + 1)),x)
```

```
[Out] ((log(x - (3^(1/2)*1i)/2 - 1/2) - log(x/2 + (3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1
/2) + (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 + 1/2)^3 +
 1/2))/((((3^(1/2)*1i)/2 + 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/
2 + 1/2)^3 + 1)) + ((log(x - (3^(1/2)*1i)/2 + 1/2) - log((3^(1/2)/2 - 1i/2)
*(x^2 + 1)^(1/2) - x/2 + (3^(1/2)*x*1i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/
2)*1i)/2 - 1/2)^3 - 1/2))/((((3^(1/2)*1i)/2 - 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i
 + 4*((3^(1/2)*1i)/2 - 1/2)^3 - 1)) + ((log(x + (3^(1/2)*1i)/2 - 1/2) - log
(x/2 + (3^(1/2)/2 - 1i/2)*(x^2 + 1)^(1/2) - (3^(1/2)*x*1i)/2 + 1))*((3^(1/2
)*1i)/2 + 2*((3^(1/2)*1i)/2 - 1/2)^3 - 1/2))/((((3^(1/2)*1i)/2 - 1/2)^2 + 1
)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/2 - 1/2)^3 - 1)) + ((log(x + (3^(1/2)
*1i)/2 + 1/2) - log((3^(1/2)/2 + 1i/2)*(x^2 + 1)^(1/2) - x/2 - (3^(1/2)*x*1
i)/2 + 1))*((3^(1/2)*1i)/2 + 2*((3^(1/2)*1i)/2 + 1/2)^3 + 1/2))/((((3^(1/2)
*1i)/2 + 1/2)^2 + 1)^(1/2)*(3^(1/2)*1i + 4*((3^(1/2)*1i)/2 + 1/2)^3 + 1))
```

3.35 $\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} dx$

Optimal result

Integrand size = 30, antiderivative size = 145

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} dx = -\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\operatorname{arctanh}\left(\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} + \frac{\sqrt{b+2\sqrt{a}\sqrt{c}}\operatorname{arctanh}\left(\frac{\sqrt{b+2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}$$

```
[Out] -1/4*arctanh(x*(b-2*a^(1/2)*c^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(b-2*a^(1
/2)*c^(1/2))^(1/2)/d/a^(1/2)/c^(1/2)+1/4*arctanh(x*(b+2*a^(1/2)*c^(1/2))^(1
/2)/(c*x^4+b*x^2+a)^(1/2))*(b+2*a^(1/2)*c^(1/2))^(1/2)/d/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2096, 1107, 214}

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4} dx = \frac{\sqrt{2\sqrt{a}\sqrt{c}+b}\operatorname{arctanh}\left(\frac{x\sqrt{2\sqrt{a}\sqrt{c}+b}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} - \frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\operatorname{arctanh}\left(\frac{x\sqrt{b-2\sqrt{a}\sqrt{c}}}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}$$

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(a*d - c*d*x^4),x]
```

```
[Out] -1/4*(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*ArcTanh[(Sqrt[b - 2*Sqrt[a]*Sqrt[c]]*x)/S
qrt[a + b*x^2 + c*x^4]])/(Sqrt[a]*Sqrt[c]*d) + (Sqrt[b + 2*Sqrt[a]*Sqrt[c]]
```

```
*ArcTanh[(Sqrt[b + 2*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[
a]*Sqrt[c]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 2096

```
Int[Sqrt[v_]/((d_) + (e_.)*(x_)^4), x_Symbol] :> With[{a = Coeff[v, x, 0],
b = Coeff[v, x, 2], c = Coeff[v, x, 4]}, Dist[a/d, Subst[Int[1/(1 - 2*b*x^2
 + (b^2 - 4*a*c)*x^4), x], x, x/Sqrt[v]], x] /; EqQ[c*d + a*e, 0] && PosQ[a
*c]] /; FreeQ[{d, e}, x] && PolyQ[v, x^2, 2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{1-2bx^2+(b^2-4ac)x^4}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)}{d} \\
&= \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{-b-2\sqrt{a}\sqrt{c}+(b^2-4ac)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} \\
&\quad - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{-b+2\sqrt{a}\sqrt{c}+(b^2-4ac)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} \\
&= -\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{b-2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d} + \frac{\sqrt{b+2\sqrt{a}\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{b+2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = \frac{-\sqrt{-b-2\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b-2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)+\sqrt{-b+2\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b+2\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\sqrt{c}d}$$

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(a*d - c*d*x^4),x]
```

```
[Out] (-(Sqrt[-b - 2*Sqrt[a]*Sqrt[c]]*ArcTan[(Sqrt[-b - 2*Sqrt[a]*Sqrt[c]]*x)/Sqr
t[a + b*x^2 + c*x^4]]) + Sqrt[-b + 2*Sqrt[a]*Sqrt[c]]*ArcTan[(Sqrt[-b + 2*S
qrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 + c*x^4]])/(4*Sqrt[a]*Sqrt[c]*d)
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{\left(2\sqrt{ac}+b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{-2\sqrt{ac}-b}}\right)+\sqrt{2\sqrt{ac}-b}\,\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{2\sqrt{ac}-b}}\right)\sqrt{-2\sqrt{ac}-b}}{4\sqrt{-2\sqrt{ac}-b}\,\sqrt{ac}\,d}$	126
default	$\frac{\left(-\frac{\left(2\sqrt{ac}+b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{-4\sqrt{ac}-2b}}\right)}{2\sqrt{ac}\,\sqrt{-4\sqrt{ac}-2b}}-\frac{\left(2\sqrt{ac}-b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{4\sqrt{ac}-2b}}\right)}{2\sqrt{ac}\,\sqrt{4\sqrt{ac}-2b}}\right)\sqrt{2}}{2d}$	140
elliptic	$\frac{2\left(-\frac{\left(2\sqrt{ac}+b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{-4\sqrt{ac}-2b}}\right)}{8\sqrt{ac}\,\sqrt{-4\sqrt{ac}-2b}}-\frac{\left(2\sqrt{ac}-b\right)\arctan\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{2}}{x\sqrt{4\sqrt{ac}-2b}}\right)}{8\sqrt{ac}\,\sqrt{4\sqrt{ac}-2b}}\right)\sqrt{2}}{d}$	140

```
[In] int((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-2*(a*c)^(1/2)-b)^(1/2)/(a*c)^(1/2)*((2*(a*c)^(1/2)+b)*arctan((c*x^4+
b*x^2+a)^(1/2)/x/(-2*(a*c)^(1/2)-b)^(1/2))+(2*(a*c)^(1/2)-b)^(1/2)*arctan((
c*x^4+b*x^2+a)^(1/2)/x/(2*(a*c)^(1/2)-b)^(1/2))*(-2*(a*c)^(1/2)-b)^(1/2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(105) = 210.

Time = 1.78 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx$$
$$=\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}+x^2\right)+\left(acd^3x^3\sqrt{\frac{1}{acd^4}}+adx\right)\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}}{cx^4-a}\right)$$
$$-\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}+x^2\right)-\left(acd^3x^3\sqrt{\frac{1}{acd^4}}+adx\right)\sqrt{\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}+b}{acd^2}}}{cx^4-a}\right)$$
$$+\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}-x^2\right)+\left(acd^3x^3\sqrt{\frac{1}{acd^4}}-adx\right)\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}}{cx^4-a}\right)$$
$$-\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{cx^4+bx^2+a}\left(ad^2\sqrt{\frac{1}{acd^4}}-x^2\right)-\left(acd^3x^3\sqrt{\frac{1}{acd^4}}-adx\right)\sqrt{-\frac{2\,acd^2\sqrt{\frac{1}{acd^4}}-b}{acd^2}}}{cx^4-a}\right)$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(c*x^4 + b*x
^2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) + x^2) + (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) +
a*d*x)*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 - a)) - 1/
8*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(c*x^4 + b*x^2
 + a)*(a*d^2*sqrt(1/(a*c*d^4)) + x^2) - (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) + a*
d*x)*sqrt((2*a*c*d^2*sqrt(1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4 - a)) + 1/8*
sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(c*x^4 + b*x^2
 + a)*(a*d^2*sqrt(1/(a*c*d^4)) - x^2) + (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) - a*
d*x)*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 - a)) - 1/8
*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(c*x^4 + b*x^
2 + a)*(a*d^2*sqrt(1/(a*c*d^4)) - x^2) - (a*c*d^3*x^3*sqrt(1/(a*c*d^4)) - a
*d*x)*sqrt(-(2*a*c*d^2*sqrt(1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 - a))
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = -\frac{\int \frac{\sqrt{a+bx^2+cx^4}}{-a+cx^4}\,dx}{d}$$

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(-c*d*x**4+a*d),x)
[Out] -Integral(sqrt(a + b*x**2 + c*x**4)/(-a + c*x**4), x)/d
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = \int -\frac{\sqrt{cx^4+bx^2+a}}{cdx^4-ad}\,dx$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 - a*d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = \int -\frac{\sqrt{cx^4+bx^2+a}}{cdx^4-ad}\,dx$$

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/(-c*d*x^4+a*d),x, algorithm="giac")
[Out] integrate(-sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 - a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{ad-cdx^4}\,dx = \int \frac{\sqrt{c\,x^4+b\,x^2+a}}{a\,d-c\,d\,x^4}\,dx$$

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(a*d - c*d*x^4),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(a*d - c*d*x^4), x)
```

3.36 $\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 239

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4} \, dx = -\frac{\sqrt{b+\sqrt{b^2+4ac}}\arctan\left(\frac{\sqrt{b+\sqrt{b^2+4ac}}x\left(b-\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d} + \frac{\sqrt{-b+\sqrt{b^2+4ac}}\operatorname{arctanh}\left(\frac{\sqrt{-b+\sqrt{b^2+4ac}}x\left(b+\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d}$$

```
[Out] 1/4*arctanh(1/4*x*(b-2*c*x^2+(4*a*c+b^2)^(1/2))*(-b+(4*a*c+b^2)^(1/2))^(1/2
)*2^(1/2)/a^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2))*(-b+(4*a*c+b^2)^(1/2))^(1
/2)/d*2^(1/2)/a^(1/2)/c^(1/2)-1/4*arctan(1/4*x*(b-2*c*x^2-(4*a*c+b^2)^(1/2)
)*(b+(4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2
))*(b+(4*a*c+b^2)^(1/2))^(1/2)/d*2^(1/2)/a^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2097}

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4} \, dx = \frac{\sqrt{\sqrt{4ac+b^2}-b}\operatorname{arctanh}\left(\frac{x\sqrt{\sqrt{4ac+b^2}-b}\left(\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d} - \frac{\sqrt{\sqrt{4ac+b^2}+b}\arctan\left(\frac{x\sqrt{\sqrt{4ac+b^2}+b}\left(-\sqrt{4ac+b^2}+b-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d}$$

```
[In] Int[Sqrt[a + b*x^2 - c*x^4]/(a*d + c*d*x^4),x]
```

```
[Out] -1/2*(Sqrt[b + Sqrt[b^2 + 4*a*c]]*ArcTan[(Sqrt[b + Sqrt[b^2 + 4*a*c]]*x*(b
- Sqrt[b^2 + 4*a*c] - 2*c*x^2))/(2*Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 -
 c*x^4])])/(Sqrt[2]*Sqrt[a]*Sqrt[c]*d) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*ArcT
anh[(Sqrt[-b + Sqrt[b^2 + 4*a*c]]*x*(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2))/(2*S
qrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(2*Sqrt[2]*Sqrt[a]*Sqrt[c
]*d)
```

Rule 2097

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^4), x_Symbo
l] :> With[{q = Sqrt[b^2 - 4*a*c]}, Simp[(-a)*(Sqrt[b + q]/(2*Sqrt[2]*Rt[(-
a)*c, 2]*d))*ArcTan[Sqrt[b + q]*x*((b - q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x] + Simp[a*(Sqrt[-b + q]/(2*Sqrt[2]*Rt[(-a)
*c, 2]*d))*ArcTanh[Sqrt[-b + q]*x*((b + q + 2*c*x^2)/(2*Sqrt[2]*Rt[(-a)*c,
2]*Sqrt[a + b*x^2 + c*x^4]))], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d
+ a*e, 0] && NegQ[a*c]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{b+\sqrt{b^2+4ac}}\tan^{-1}\left(\frac{\sqrt{b+\sqrt{b^2+4ac}}x\left(b-\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d} + \frac{\sqrt{-b+\sqrt{b^2+4ac}}\tanh^{-1}\left(\frac{\sqrt{-b+\sqrt{b^2+4ac}}x\left(b+\sqrt{b^2+4ac}-2cx^2\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{2}\sqrt{a}\sqrt{c}d}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \frac{i\left(\sqrt{-b-2i\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b-2i\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2-cx^4}}\right)-\sqrt{-b+2i\sqrt{a}\sqrt{c}}\arctan\left(\frac{\sqrt{-b+2i\sqrt{a}\sqrt{c}}x}{\sqrt{a+bx^2-cx^4}}\right)\right)}{4\sqrt{a}\sqrt{c}d}$$

```
[In] Integrate[Sqrt[a + b*x^2 - c*x^4]/(a*d + c*d*x^4),x]
```

```
[Out] ((I/4)*(Sqrt[-b - (2*I)*Sqrt[a]*Sqrt[c]]*ArcTan[(Sqrt[-b - (2*I)*Sqrt[a]*Sq
rt[c]]*x)/Sqrt[a + b*x^2 - c*x^4]] - Sqrt[-b + (2*I)*Sqrt[a]*Sqrt[c]]*ArcTa
n[(Sqrt[-b + (2*I)*Sqrt[a]*Sqrt[c]]*x)/Sqrt[a + b*x^2 - c*x^4]]))/(Sqrt[a]*
Sqrt[c]*d)
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.47

method	result
pseudoelliptic	$-\frac{\sqrt{2\sqrt{4ac+b^2}+2b}\left(b-\sqrt{4ac+b^2}\right)\left(\ln\left(\frac{-c\,x^4+\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2\sqrt{4ac+b^2}+2b}\,x+\sqrt{4ac+b^2}\,x^2+b\,x^2+a}{x^2}\right)-\ln\left(\frac{c\,x^4+\sqrt{-c\,x^4+b}}{}\right.\right.}{}$
elliptic	$\frac{\left(\frac{\sqrt{b+\sqrt{4ac+b^2}}\,b\ln\left(\frac{-c\,x^4+b\,x^2+a}{x^2}-\frac{\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2}\,\sqrt{b+\sqrt{4ac+b^2}}}{x}+\sqrt{4ac+b^2}\right)}{16dac}+\frac{b^2\arctan\left(\frac{\frac{2\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2}}{x}-2\sqrt{b+\sqrt{4ac}}}{2\sqrt{-b+\sqrt{4ac+b^2}}}\right.}{8dac\sqrt{-b+\sqrt{4ac+b^2}}}\right.}{}$
default	$\frac{\left(\frac{\sqrt{b+\sqrt{4ac+b^2}}\,b\ln\left(\frac{-c\,x^4+b\,x^2+a}{x^2}-\frac{\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2}\,\sqrt{b+\sqrt{4ac+b^2}}}{x}+\sqrt{4ac+b^2}\right)}{16ac}+\frac{\left(b+\sqrt{4ac+b^2}\right)b\arctan\left(\frac{\frac{2\sqrt{-c\,x^4+b\,x^2+a}\,\sqrt{2}}{x}}{2\sqrt{-b+\sqrt{4}}}\right.}{8ac\sqrt{-b+\sqrt{4ac+b^2}}}\right.}{}$

```
[In] int((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/(2*(4*a*c+b^2)^(1/2)-2*b)^(1/2)*((2*(4*a*c+b^2)^(1/2)+2*b)^(1/2)*(b-(
4*a*c+b^2)^(1/2))*(ln((-c*x^4+(-c*x^4+b*x^2+a)^(1/2)*(2*(4*a*c+b^2)^(1/2)+2
*b)^(1/2)*x+(4*a*c+b^2)^(1/2)*x^2+b*x^2+a)/x^2)-ln((c*x^4+(-c*x^4+b*x^2+a)^
(1/2)*(2*(4*a*c+b^2)^(1/2)+2*b)^(1/2)*x-(4*a*c+b^2)^(1/2)*x^2-b*x^2-a)/x^2)
)*(2*(4*a*c+b^2)^(1/2)-2*b)^(1/2)-16*a*c*(arctan((((2*(4*a*c+b^2)^(1/2)+2*b)
^(1/2)*x-2*(-c*x^4+b*x^2+a)^(1/2))/x/(2*(4*a*c+b^2)^(1/2)-2*b)^(1/2))-arcta
n(((2*(4*a*c+b^2)^(1/2)+2*b)^(1/2)*x+2*(-c*x^4+b*x^2+a)^(1/2))/x/(2*(4*a*c+
b^2)^(1/2)-2*b)^(1/2))))/d/a/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. 2(187) = 374.

Time = 1.68 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = -\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}+\sqrt{-cx^4+bx^2+a}x^2+\left(acd^3x^3\sqrt{-\frac{1}{acd^4}}-\right.}{cx^4+a}\right.$$

$$+\frac{1}{8}\sqrt{\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}-b}{acd^2}}\log\left(-\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}+\sqrt{-cx^4+bx^2+a}x^2-\left(acd^3x^3\sqrt{-\frac{1}{acd^4}}-\right.}{cx^4+a}\right.$$

$$-\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}-\sqrt{-cx^4+bx^2+a}x^2+\left(acd^3x^3\sqrt{-\frac{1}{acd^4}}+\right.}{cx^4+a}\right.$$

$$+\frac{1}{8}\sqrt{-\frac{2\,acd^2\sqrt{-\frac{1}{acd^4}}+b}{acd^2}}\log\left(\frac{\sqrt{-cx^4+bx^2+a}ad^2\sqrt{-\frac{1}{acd^4}}-\sqrt{-cx^4+bx^2+a}x^2-\left(acd^3x^3\sqrt{-\frac{1}{acd^4}}+\right.}{cx^4+a}\right.$$

```
[In] integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)/(a*c*d^2))*log(-(sqrt(-c*x^4 +
 b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) + sqrt(-c*x^4 + b*x^2 + a)*x^2 + (a*c*
d^3*x^3*sqrt(-1/(a*c*d^4)) - a*d*x)*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)
/(a*c*d^2)))/(c*x^4 + a)) + 1/8*sqrt((2*a*c*d^2*sqrt(-1/(a*c*d^4)) - b)/(a*
c*d^2))*log(-(sqrt(-c*x^4 + b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) + sqrt(-c*x
^4 + b*x^2 + a)*x^2 - (a*c*d^3*x^3*sqrt(-1/(a*c*d^4)) - a*d*x)*sqrt((2*a*c*
d^2*sqrt(-1/(a*c*d^4)) - b)/(a*c*d^2)))/(c*x^4 + a)) - 1/8*sqrt(-(2*a*c*d^2
*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(-c*x^4 + b*x^2 + a)*a*d^2*sqr
t(-1/(a*c*d^4)) - sqrt(-c*x^4 + b*x^2 + a)*x^2 + (a*c*d^3*x^3*sqrt(-1/(a*c*
d^4)) + a*d*x)*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2)))/(c*x^4
+ a)) + 1/8*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^4)) + b)/(a*c*d^2))*log((sqrt(-
c*x^4 + b*x^2 + a)*a*d^2*sqrt(-1/(a*c*d^4)) - sqrt(-c*x^4 + b*x^2 + a)*x^2
- (a*c*d^3*x^3*sqrt(-1/(a*c*d^4)) + a*d*x)*sqrt(-(2*a*c*d^2*sqrt(-1/(a*c*d^
4)) + b)/(a*c*d^2)))/(c*x^4 + a))
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \frac{\int \frac{\sqrt{a+bx^2-cx^4}}{a+cx^4}\,dx}{d}$$

```
[In] integrate((-c*x**4+b*x**2+a)**(1/2)/(c*d*x**4+a*d),x)
[Out] Integral(sqrt(a + b*x**2 - c*x**4)/(a + c*x**4), x)/d
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \int \frac{\sqrt{-cx^4+bx^2+a}}{cdx^4+ad}\,dx$$

```
[In] integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="maxima")
[Out] integrate(sqrt(-c*x^4 + b*x^2 + a)/(c*d*x^4 + a*d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \int \frac{\sqrt{-cx^4+bx^2+a}}{cdx^4+ad}\,dx$$

```
[In] integrate((-c*x^4+b*x^2+a)^(1/2)/(c*d*x^4+a*d),x, algorithm="giac")
[Out] integrate(sqrt(-c*x^4 + b*x^2 + a)/(c*d*x^4 + a*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2-cx^4}}{ad+cdx^4}\,dx = \int \frac{\sqrt{-c\,x^4+b\,x^2+a}}{c\,d\,x^4+a\,d}\,dx$$

```
[In] int((a + b*x^2 - c*x^4)^(1/2)/(a*d + c*d*x^4),x)
[Out] int((a + b*x^2 - c*x^4)^(1/2)/(a*d + c*d*x^4), x)
```

3.37 $\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 309

$$\begin{aligned}&\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx\\&=\frac{e(12bcd-16ad^2-7be^2)\,(e+2dx)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{128d^4\,(a+bx^2)}\\&\quad+\frac{bx^2(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}\\&\quad-\frac{(32bcd-80ad^2-35be^2+42bdex)\,(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{240d^3\,(a+bx^2)}\\&\quad+\frac{e(4cd-e^2)\,(12bcd-16ad^2-7be^2)\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{256d^{9/2}\,(a+bx^2)}\end{aligned}$$

```
[Out] 1/5*b*x^2*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1/240*(42*b*d
*e*x-80*a*d^2+32*b*c*d-35*b*e^2)*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d^
3/(b*x^2+a)+1/256*e*(4*c*d-e^2)*(-16*a*d^2+12*b*c*d-7*b*e^2)*arctanh(1/2*(2
*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(9/2)/(b*x^2+a)+
1/128*e*(-16*a*d^2+12*b*c*d-7*b*e^2)*(2*d*x+e)*(d*x^2+e*x+c)^(1/2)*((b*x^2+
a)^2)^(1/2)/d^4/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6876, 1667, 793, 626, 635, 212}

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}(4cd-e^2)\left(-16ad^2+12bcd-7be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{256d^{9/2}\left(a+bx^2\right)}$$
$$+ \frac{e\sqrt{a^2+2abx^2+b^2x^4}(2dx+e)\sqrt{c+dx^2+ex}(-16ad^2+12bcd-7be^2)}{128d^4\left(a+bx^2\right)}$$
$$- \frac{\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}\left(-80ad^2+32bcd+42bdex-35be^2\right)}{240d^3\left(a+bx^2\right)}$$
$$+ \frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{5d\left(a+bx^2\right)}$$

```
[In] Int[x*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*(12*b*c*d - 16*a*d^2 - 7*b*e^2)*(e + 2*d*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(128*d^4*(a + b*x^2)) + (b*x^2*(c + e*x + d*x^2)
^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2)) - ((32*b*c*d - 80
*a*d^2 - 35*b*e^2 + 42*b*d*e*x)*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(240*d^3*(a + b*x^2)) + (e*(4*c*d - e^2)*(12*b*c*d - 16*a*d^2
 - 7*b*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*
Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 6876

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x(2ab+2b^2x^2)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x^2}\\
&= \frac{bx^2(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}\\
&\quad+\frac{\sqrt{a^2+2abx^2+b^2x^4}\int x(-2b(2bc-5ad)-7b^2ex)\sqrt{c+ex+dx^2}\,dx}{5d\,(2ab+2b^2x^2)}\\
&= \frac{bx^2(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}\\
&\quad-\frac{(32bcd-80ad^2-35be^2+42bdex)\,(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{240d^3\,(a+bx^2)}\\
&\quad+\frac{\left(be(12bcd-16ad^2-7be^2)\sqrt{a^2+2abx^2+b^2x^4}\right)\int\sqrt{c+ex+dx^2}\,dx}{16d^3\,(2ab+2b^2x^2)}
\end{aligned}
$$

$$= \frac{e(12bcd - 16ad^2 - 7be^2)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{128d^4(a + bx^2)} + \frac{bx^2(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{5d(a + bx^2)} - \frac{(32bcd - 80ad^2 - 35be^2 + 42bdex)(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{240d^3(a + bx^2)} + \frac{\left(be(4cd - e^2)(12bcd - 16ad^2 - 7be^2)\sqrt{a^2 + 2abx^2 + b^2x^4}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{128d^4(2ab + 2b^2x^2)}$$

$$= \frac{e(12bcd - 16ad^2 - 7be^2)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{128d^4(a + bx^2)} + \frac{bx^2(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{5d(a + bx^2)} - \frac{(32bcd - 80ad^2 - 35be^2 + 42bdex)(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{240d^3(a + bx^2)} + \frac{\left(be(4cd - e^2)(12bcd - 16ad^2 - 7be^2)\sqrt{a^2 + 2abx^2 + b^2x^4}\right)\text{Subst}\left(\int \frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{64d^4(2ab + 2b^2x^2)}$$

$$= \frac{e(12bcd - 16ad^2 - 7be^2)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{128d^4(a + bx^2)} + \frac{bx^2(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{5d(a + bx^2)} - \frac{(32bcd - 80ad^2 - 35be^2 + 42bdex)(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{240d^3(a + bx^2)} + \frac{e(4cd - e^2)(12bcd - 16ad^2 - 7be^2)\sqrt{a^2 + 2abx^2 + b^2x^4}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{256d^{9/2}(a + bx^2)}$$

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.69

$$\int x\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}\,dx$$

$$= \frac{\sqrt{(a + bx^2)^2}\left(2\sqrt{d}\sqrt{c + x(e + dx)}(80ad^2(8cd - 3e^2 + 2dex + 8d^2x^2) + b(-256c^2d^2 - 105e^4 + 70de^3x - \right.}{}$$

```
[In] Integrate[x*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(80*a*d^2*(8*c*d - 3*
e^2 + 2*d*e*x + 8*d^2*x^2) + b*(-256*c^2*d^2 - 105*e^4 + 70*d*e^3*x - 56*d^
```

```
2*e^2*x^2 + 48*d^3*e*x^3 + 384*d^4*x^4 + 4*c*d*(115*e^2 - 58*d*e*x + 32*d^2
*x^2))) - 15*e*(-4*c*d + e^2)*(-12*b*c*d + 16*a*d^2 + 7*b*e^2)*Log[e + 2*d*
x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]]))/(3840*d^(9/2)*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.79

method	result
risch	$\frac{(384b\,x^4d^4+48e\,x^3b\,d^3+640a\,d^4x^2+128bc\,d^3x^2-56b\,d^2e^2x^2+160a\,d^3ex-232bc\,d^2ex+70bd\,e^3x+640ac\,d^3-240e^2d^2a-256b\,c^2d^2+460d}{1920d^4(b\,x^2+a)}$
default	$\frac{\sqrt{(b\,x^2+a)^2}\left(768d^{\frac{9}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}b\,x^2-672d^{\frac{7}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}bex+1280d^{\frac{9}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}a-512d^{\frac{7}{2}}(d\,x^2+ex+c)^{\frac{3}{2}}bc+560d^{\frac{5}{2}}(d\,x^2+\right.}{}$

```
[In] int(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1920*(384*b*d^4*x^4+48*b*d^3*e*x^3+640*a*d^4*x^2+128*b*c*d^3*x^2-56*b*d^2
*e^2*x^2+160*a*d^3*e*x-232*b*c*d^2*e*x+70*b*d*e^3*x+640*a*c*d^3-240*a*d^2*e
^2-256*b*c^2*d^2+460*b*c*d*e^2-105*b*e^4)*(d*x^2+e*x+c)^(1/2)/d^4*((b*x^2+a
)^2)^(1/2)/(b*x^2+a)-1/256*e*(64*a*c*d^3-16*a*d^2*e^2-48*b*c^2*d^2+40*b*c*d
*e^2-7*b*e^4)/d^(9/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x^2+a
)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.52

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\left[\frac{15\,(7\,be^5-8\,(5\,bcd-2\,ad^2)e^3+16\,(3\,bc^2d^2-4\,acd^3)e)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+\right.}{}\right.$$
$$-\frac{15\,(7\,be^5-8\,(5\,bcd-2\,ad^2)e^3+16\,(3\,bc^2d^2-4\,acd^3)e)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-2\,(384\,bd^5x}{}$$

```
[In] integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b*e^5 - 8*(5*b*c*d - 2*a*d^2)*e^3 + 16*(3*b*c^2*d^2 - 4*a*c*
d^3)*e)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(d) + 4*c*d + e^2) + 4*(384*b*d^5*x^4 + 48*b*d^4*e*x^3 - 256*b*c^2*d
^3 + 640*a*c*d^4 - 105*b*d*e^4 + 20*(23*b*c*d^2 - 12*a*d^3)*e^2 + 8*(16*b*c
```

```
*d^4 + 80*a*d^5 - 7*b*d^3*e^2)*x^2 + 2*(35*b*d^2*e^3 - 4*(29*b*c*d^3 - 20*a
*d^4)*e)*x)*sqrt(d*x^2 + e*x + c))/d^5, -1/3840*(15*(7*b*e^5 - 8*(5*b*c*d -
 2*a*d^2)*e^3 + 16*(3*b*c^2*d^2 - 4*a*c*d^3)*e)*sqrt(-d)*arctan(1/2*sqrt(d*
x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 2*(384*b*d^5
*x^4 + 48*b*d^4*e*x^3 - 256*b*c^2*d^3 + 640*a*c*d^4 - 105*b*d*e^4 + 20*(23*
b*c*d^2 - 12*a*d^3)*e^2 + 8*(16*b*c*d^4 + 80*a*d^5 - 7*b*d^3*e^2)*x^2 + 2*(
35*b*d^2*e^3 - 4*(29*b*c*d^3 - 20*a*d^4)*e)*x)*sqrt(d*x^2 + e*x + c))/d^5]
```

Sympy [F]

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\sqrt{c+dx^2+ex}\sqrt{\left(a+bx^2\right)^2}\,dx$$

```
[In] integrate(x*(d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
[Out] Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.16

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\frac{1}{1920}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\left(8\,bx\text{sgn}\left(bx^2+a\right)+\frac{be\text{sgn}(bx^2+a)}{d}\right)x+\frac{16\,bcd^3\text{sgn}(bx^2+a)+80\,ad^4\text{sgr}}{c}\right.\right.\right.$$
$$-\frac{\left(48\,bc^2d^2e\text{sgn}(bx^2+a)-64\,acd^3e\text{sgn}(bx^2+a)-40\,bcde^3\text{sgn}(bx^2+a)+16\,ad^2e^3\text{sgn}(bx^2+a)+7\,be^5\text{s}\right.}{256\,d^{\frac{9}{2}}}$$

```
[In] integrate(x*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(d*x^2 + e*x + c)*(2*(4*(6*(8*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2
 + a)/d)*x + (16*b*c*d^3*sgn(b*x^2 + a) + 80*a*d^4*sgn(b*x^2 + a) - 7*b*d^2
*e^2*sgn(b*x^2 + a))/d^4)*x - (116*b*c*d^2*e*sgn(b*x^2 + a) - 80*a*d^3*e*sg
n(b*x^2 + a) - 35*b*d*e^3*sgn(b*x^2 + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x^2
 + a) - 640*a*c*d^3*sgn(b*x^2 + a) - 460*b*c*d*e^2*sgn(b*x^2 + a) + 240*a*d
^2*e^2*sgn(b*x^2 + a) + 105*b*e^4*sgn(b*x^2 + a))/d^4) - 1/256*(48*b*c^2*d^
2*e*sgn(b*x^2 + a) - 64*a*c*d^3*e*sgn(b*x^2 + a) - 40*b*c*d*e^3*sgn(b*x^2 +
 a) + 16*a*d^2*e^3*sgn(b*x^2 + a) + 7*b*e^5*sgn(b*x^2 + a))*log(abs(2*(sqrt
(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int x\,\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

```
[In] int(x*((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.38 $\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 283

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = -\frac{(4bcd-16ad^2-5be^2)\,(e+2dx)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{64d^3\,(a+bx^2)} - \frac{5be(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{24d^2\,(a+bx^2)} + \frac{bx(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{(4cd-e^2)\,(4bcd-16ad^2-5be^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{128d^{7/2}\,(a+bx^2)}$$

```
[Out] -5/24*b*e*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)+1/4*b*x*(d*
x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1/128*(4*c*d-e^2)*(-16*a*d
^2+4*b*c*d-5*b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*
x^2+a)^2)^(1/2)/d^(7/2)/(b*x^2+a)-1/64*(-16*a*d^2+4*b*c*d-5*b*e^2)*(2*d*x+e
)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^3/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used

= {6876, 1675, 654, 626, 635, 212}

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$

$$= -\frac{\sqrt{a^2+2abx^2+b^2x^4}(4cd-e^2)(-16ad^2+4bcd-5be^2)\,\mathrm{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx^2)}$$
$$-\frac{5be\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{24d^2(a+bx^2)}$$
$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}(2dx+e)\sqrt{c+dx^2+ex}(-16ad^2+4bcd-5be^2)}{64d^3(a+bx^2)}$$
$$+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{4d(a+bx^2)}$$

```
[In] Int[Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/64*((4*b*c*d - 16*a*d^2 - 5*b*e^2)*(e + 2*d*x)*Sqrt[c + e*x + d*x^2]*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b*x^2)) - (5*b*e*(c + e*x + d*x^2)^
(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*d^2*(a + b*x^2)) + (b*x*(c + e*x
 + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a + b*x^2)) - ((4*c*
d - e^2)*(4*b*c*d - 16*a*d^2 - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Arc
Tanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x^
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1675

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 6876

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(2ab+2b^2x^2\right)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x^2}\\
&= \frac{bx(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\left(a+bx^2\right)}\\
&\quad+\frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(-2b(bc-4ad)-5b^2ex\right)\sqrt{c+ex+dx^2}\,dx}{4d\left(2ab+2b^2x^2\right)}\\
&= -\frac{5be(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{24d^2\left(a+bx^2\right)}\\
&\quad+\frac{bx(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\left(a+bx^2\right)}\\
&\quad+\frac{\left(\left(-4bd(bc-4ad)+5b^2e^2\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \sqrt{c+ex+dx^2}\,dx}{8d^2\left(2ab+2b^2x^2\right)}\\
&= -\frac{\left(4bcd-16ad^2-5be^2\right)(e+2dx)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{64d^3\left(a+bx^2\right)}\\
&\quad-\frac{5be(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{24d^2\left(a+bx^2\right)}\\
&\quad+\frac{bx(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\left(a+bx^2\right)}\\
&\quad+\frac{\left(\left(4cd-e^2\right)\left(-4bd(bc-4ad)+5b^2e^2\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{64d^3\left(2ab+2b^2x^2\right)}
\end{aligned}
$$

$$= -\frac{(4bcd - 16ad^2 - 5be^2)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{64d^3(a + bx^2)} - \frac{5be(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{24d^2(a + bx^2)} + \frac{bx(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4d(a + bx^2)} + \frac{\left((4cd - e^2)(-4bd(bc - 4ad) + 5b^2e^2)\sqrt{a^2 + 2abx^2 + b^2x^4}\right)\text{Subst}\left(\int \frac{1}{4d - x^2}\,dx, x, \frac{e + 2dx}{\sqrt{c + ex + dx^2}}\right)}{32d^3(2ab + 2b^2x^2)}$$

$$= -\frac{(4bcd - 16ad^2 - 5be^2)(e + 2dx)\sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{64d^3(a + bx^2)} - \frac{5be(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{24d^2(a + bx^2)} + \frac{bx(c + ex + dx^2)^{3/2}\sqrt{a^2 + 2abx^2 + b^2x^4}}{4d(a + bx^2)} - \frac{(4cd - e^2)(4bcd - 16ad^2 - 5be^2)\sqrt{a^2 + 2abx^2 + b^2x^4}\tanh^{-1}\left(\frac{e + 2dx}{2\sqrt{d}\sqrt{c + ex + dx^2}}\right)}{128d^{7/2}(a + bx^2)}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.76

$$\int \sqrt{c + ex + dx^2}\sqrt{a^2 + 2abx^2 + b^2x^4}\,dx = \frac{\sqrt{(a + bx^2)^2}\left(\sqrt{d}\sqrt{c + x(e + dx)}(48ad^2(e + 2dx) + b(15e^3 - 10de^2x + 8d^2ex^2 + 48d^3x^3 + 4cd(-13e + 6d\right.}{192d^{7/2}}$$

```
[In] Integrate[Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*Sqrt[c + x*(e + d*x)]*(48*a*d^2*(e + 2*d*x) +
 b*(15*e^3 - 10*d*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x))
) + 3*(16*b*c^2*d^2 + 16*a*d^2*e^2 + 5*b*e^4)*ArcTanh[(Sqrt[d]*x)/(Sqrt[c]
- Sqrt[c + x*(e + d*x)])] + 24*c*d*(8*a*d^2 + 3*b*e^2)*ArcTanh[(Sqrt[d]*x)/
(-Sqrt[c] + Sqrt[c + x*(e + d*x)])]))/(192*d^(7/2)*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.67

method	result
risch	$\frac{\left(48 b\,x^{3} d^{3}+8 e\,x^{2} b\,d^{2}+96 a\,d^{3} x+24 b c\,d^{2} x-10 b d\,e^{2} x+48 a\,d^{2} e-52 b c d e+15 b\,e^{3}\right) \sqrt{d\,x^{2}+e x+c}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{192 d^{3} \left(b\,x^{2}+a\right)}+\frac{\left(64 a c\,d^{3}-16 e^{2} d^{2} a-16 b\right.}{}$
default	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(96 d^{\frac{7}{2}} \left(d\,x^{2}+e x+c\right)^{\frac{3}{2}} b x-80 d^{\frac{5}{2}} \left(d\,x^{2}+e x+c\right)^{\frac{3}{2}} b e+192 d^{\frac{9}{2}} \sqrt{d\,x^{2}+e x+c}\,a x-48 d^{\frac{7}{2}} \sqrt{d\,x^{2}+e x+c}\,b c x+60 d^{\frac{5}{2}} \sqrt{d\,x^{2}+e x+c}\,b\right.}{}$

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(48*b*d^3*x^3+8*b*d^2*e*x^2+96*a*d^3*x+24*b*c*d^2*x-10*b*d*e^2*x+48*a
*d^2*e-52*b*c*d*e+15*b*e^3)*(d*x^2+e*x+c)^(1/2)/d^3*((b*x^2+a)^2)^(1/2)/(b*
x^2+a)+1/128*(64*a*c*d^3-16*a*d^2*e^2-16*b*c^2*d^2+24*b*c*d*e^2-5*b*e^4)/d^
(7/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^
2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.28

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$=\left[\frac{3\left(16\,bc^2d^2-64\,acd^3+5\,be^4-8\,(3\,bcd-2\,ad^2)e^2\right)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{\right.}{}\right.$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*b*c^2*d^2 - 64*a*c*d^3 + 5*b*e^4 - 8*(3*b*c*d - 2*a*d^2)*e^2)
*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt
(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 8*b*d^3*e*x^2 + 15*b*d*e^3 - 4*(13*b
*c*d^2 - 12*a*d^3)*e + 2*(12*b*c*d^3 + 48*a*d^4 - 5*b*d^2*e^2)*x)*sqrt(d*x^
2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 - 64*a*c*d^3 + 5*b*e^4 - 8*(3*b*c
*d - 2*a*d^2)*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sq
rt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x^3 + 8*b*d^3*e*x^2 + 15*b*d*
e^3 - 4*(13*b*c*d^2 - 12*a*d^3)*e + 2*(12*b*c*d^3 + 48*a*d^4 - 5*b*d^2*e^2)
*x)*sqrt(d*x^2 + e*x + c))/d^4]
```

Sympy [F]

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \int \sqrt{c+dx^2+ex}\sqrt{(a+bx^2)^2}\,dx$$

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.93

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$$
$$= \frac{1}{192}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{be\mathrm{sgn}\left(bx^2+a\right)}{d}\right)x+\frac{12\,bcd^2\mathrm{sgn}\left(bx^2+a\right)+48\,ad^3\mathrm{sgn}\left(bx^2\right.}{d^3}\right.\right.$$
$$+\frac{\left(16\,bc^2d^2\mathrm{sgn}\left(bx^2+a\right)-64\,acd^3\mathrm{sgn}\left(bx^2+a\right)-24\,bcde^2\mathrm{sgn}\left(bx^2+a\right)+16\,ad^2e^2\mathrm{sgn}\left(bx^2+a\right)+5\,be^4\mathrm{sgn}\left(\right.\right.}{128\,d^{\frac{7}{2}}}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + e*x + c)*(2*(4*(6*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2 + a
)/d)*x + (12*b*c*d^2*sgn(b*x^2 + a) + 48*a*d^3*sgn(b*x^2 + a) - 5*b*d*e^2*s
gn(b*x^2 + a))/d^3)*x - (52*b*c*d*e*sgn(b*x^2 + a) - 48*a*d^2*e*sgn(b*x^2 +
 a) - 15*b*e^3*sgn(b*x^2 + a))/d^3) + 1/128*(16*b*c^2*d^2*sgn(b*x^2 + a) -
64*a*c*d^3*sgn(b*x^2 + a) - 24*b*c*d*e^2*sgn(b*x^2 + a) + 16*a*d^2*e^2*sgn(
b*x^2 + a) + 5*b*e^4*sgn(b*x^2 + a))*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + e*
x + c))*sqrt(d) + e))/d^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx=\int \sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

```
[In] int(((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
[Out] int(((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.39 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}
&\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx \\
&= \frac{(8ad^2-be^2-2bdex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2(a+bx^2)} \\
&\quad + \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} \\
&\quad + \frac{e(8ad^2-b(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{16d^{5/2}(a+bx^2)} \\
&\quad - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{a+bx^2}
\end{aligned}$$

```
[Out] 1/3*b*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+1/16*e*(8*a*d^2-b
*(4*c*d-e^2))*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x^2+a)
^2)^(1/2)/d^(5/2)/(b*x^2+a)-a*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(
1/2))*c^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/8*(-2*b*d*e*x+8*a*d^2-b*e^2)*
(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {6876, 1667, 828, 857, 635, 212, 738}

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}(8ad^2-b(4cd-e^2))\operatorname{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx^2)} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{a+bx^2} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}(8ad^2-2bdex-be^2)}{8d^2(a+bx^2)} + \frac{b\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{3d(a+bx^2)}$$

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] ((8*a*d^2 - b*e^2 - 2*b*d*e*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(8*d^2*(a + b*x^2)) + (b*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) + (e*(8*a*d^2 - b*(4*c*d - e^2))*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x
^2])])/(16*d^(5/2)*(a + b*x^2)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 6876

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{(2ab+2b^2x^2)\sqrt{c+ex+dx^2}}{x}\,dx}{2ab+2b^2x^2}$$

$$= \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{(6abd-3b^2ex)\sqrt{c+ex+dx^2}}{x}\,dx}{3d(2ab+2b^2x^2)}$$

$$= \frac{(8ad^2-be^2-2bdex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2(a+bx^2)} + \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{-24abcd^2-\frac{3}{2}be(8ad^2-b(4cd-e^2))x}{x\sqrt{c+ex+dx^2}}\,dx}{12d^2(2ab+2b^2x^2)}$$

$$= \frac{(8ad^2-be^2-2bdex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2(a+bx^2)} + \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{\left(2abc\sqrt{a^2+2abx^2+b^2x^4}\right)\int\frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x^2} + \frac{\left(be(8ad^2-b(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\right)\int\frac{1}{\sqrt{c+ex+dx^2}}\,dx}{8d^2(2ab+2b^2x^2)}$$

$$= \frac{(8ad^2-be^2-2bdex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2(a+bx^2)} + \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} - \frac{\left(4abc\sqrt{a^2+2abx^2+b^2x^4}\right)\mathrm{Subst}\left(\int\frac{1}{4c-x^2}\,dx, x, \frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x^2} + \frac{\left(be(8ad^2-b(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\right)\mathrm{Subst}\left(\int\frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{4d^2(2ab+2b^2x^2)}$$

$$= \frac{(8ad^2-be^2-2bdex)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d^2(a+bx^2)} + \frac{b(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)} + \frac{e(8ad^2-b(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{16d^{5/2}(a+bx^2)} - \frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{a+bx^2}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$$
$$= \frac{\sqrt{(a+bx^2)^2}\left(2\sqrt{d}\sqrt{c+x(e+dx)}(24ad^2+b(8cd-3e^2+2dex+8d^2x^2))+96a\sqrt{c}d^{5/2}\text{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+x(}}{\sqrt{c}}\right.\right.}{48d^{5/2}\,(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(24*a*d^2 + b*(8*c*d
- 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 96*a*Sqrt[c]*d^(5/2)*ArcTanh[(Sqrt[d]*x -
 Sqrt[c + x*(e + d*x)])/Sqrt[c]] - 3*e*(8*a*d^2 + b*(-4*c*d + e^2))*Log[e +
 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]]))/(48*d^(5/2)*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.88

method	result
default	$-\frac{\sqrt{(b\,x^2+a)^2}\left(48\sqrt{c}\,d^{\frac{7}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)a-16d^{\frac{5}{2}}\left(d\,x^2+ex+c\right)^{\frac{3}{2}}b+12d^{\frac{5}{2}}\sqrt{d\,x^2+ex+c}\,bex-48d^{\frac{7}{2}}\sqrt{d\,x^2+ex+c}\,a+6d^{\frac{3}{2}}\sqrt{}\right.}{48(b\,x}$

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*((b*x^2+a)^2)^(1/2)*(48*c^(1/2)*d^(7/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+
e*x+c)^(1/2))/x)*a-16*d^(5/2)*(d*x^2+e*x+c)^(3/2)*b+12*d^(5/2)*(d*x^2+e*x+c
)^(1/2)*b*e*x-48*d^(7/2)*(d*x^2+e*x+c)^(1/2)*a+6*d^(3/2)*(d*x^2+e*x+c)^(1/2
)*b*e^2-24*d^3*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*a*e+
12*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*c*d^2*e-3*ln(1
/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*e^3)/(b*x^2+a)/d^(7
/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.73 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$$
$$= \left[\frac{48\,a\sqrt{c}d^3\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)+3\,(be^3-4\,(bcd-2\,ad^2)e)\sqrt{d}\log\left(8\,d^2x^2+8\,de\right.}{}\right.$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a*sqrt(c)*d^3*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e
*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*
e)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sq
rt(d) + 4*c*d + e^2) + 4*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3
- 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3, 1/48*(24*a*sqrt(c)*d^3*log((8*c*e*
x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2
)/x^2) - 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 +
 e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(8*b*d^3*x^2 +
2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3,
 1/96*(96*a*sqrt(-c)*d^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
-c)/(c*d*x^2 + c*e*x + c^2)) + 3*(b*e^3 - 4*(b*c*d - 2*a*d^2)*e)*sqrt(d)*lo
g(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d
 + e^2) + 4*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 + 24*a*d^3 - 3*b*d*e^2)*
sqrt(d*x^2 + e*x + c))/d^3, 1/48*(48*a*sqrt(-c)*d^3*arctan(1/2*sqrt(d*x^2 +
 e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 3*(b*e^3 - 4*(b*c
*d - 2*a*d^2)*e)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt
(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(8*b*d^3*x^2 + 2*b*d^2*e*x + 8*b*c*d^2 +
24*a*d^3 - 3*b*d*e^2)*sqrt(d*x^2 + e*x + c))/d^3]
```

Sympy [F]

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx^2)^2}}{x}\,dx$$

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x}\,dx$$

```
[In] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)
```

```
[Out] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)
```

3.40 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 294

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{((bc+4ad)e+2d(bc+2ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cd\,(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} + \frac{(4bcd+8ad^2-be^2)\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)} - \frac{ae\sqrt{a^2+2abx^2+b^2x^4}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{c}\,(a+bx^2)}$$

```
[Out] -a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)+1/8*(8*a*d^2+4*b*c
*d-b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x^2+a)^2)^
(1/2)/d^(3/2)/(b*x^2+a)-1/2*a*e*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)
^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/c^(1/2)+1/4*((4*a*d+b*c)*e+2*d*(2*a*d
+b*c)*x)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/d/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6876, 1664, 828, 857, 635, 212, 738}

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$$
$$=\frac{\sqrt{a^2+2abx^2+b^2x^4}(8ad^2+4bcd-be^2)\operatorname{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx^2)}-\frac{ae\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}(a+bx^2)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{cx(a+bx^2)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}(e(4ad+bc)+2dx(2ad+bc))}{4cd(a+bx^2)}$$

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (((b*c + 4*a*d)*e + 2*d*(b*c + 2*a*d)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(4*c*d*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a + b*x^2)) + ((4*b*c*d + 8*a*d^2 - b*e^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e
*x + d*x^2])])/(8*d^(3/2)*(a + b*x^2)) - (a*e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[c]*(a +
b*x^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 6876

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(2ab+2b^2x^2)\sqrt{c+ex+dx^2}}{x^2}\,dx}{2ab+2b^2x^2} \\
&= -\frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} \\
&\quad - \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(-abe-2b(bc+2ad)x)\sqrt{c+ex+dx^2}}{x}\,dx}{c\,(2ab+2b^2x^2)} \\
&= \frac{((bc+4ad)e+2d(bc+2ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cd\,(a+bx^2)} \\
&\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} \\
&\quad + \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{4abcde+bc(4bcd+8ad^2-be^2)x}{x\sqrt{c+ex+dx^2}}\,dx}{4cd\,(2ab+2b^2x^2)} \\
&= \frac{((bc+4ad)e+2d(bc+2ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cd\,(a+bx^2)} \\
&\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} \\
&\quad + \frac{\left(abe\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x^2} \\
&\quad + \frac{\left(b(4bcd+8ad^2-be^2)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{4d\,(2ab+2b^2x^2)} \\
&= \frac{((bc+4ad)e+2d(bc+2ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cd\,(a+bx^2)} \\
&\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} \\
&\quad - \frac{\left(2abe\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x^2} \\
&\quad + \frac{\left(b(4bcd+8ad^2-be^2)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{2d\,(2ab+2b^2x^2)}
\end{aligned}$$

$$= \frac{((bc+4ad)e+2d(bc+2ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cd\,(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} + \frac{(4bcd+8ad^2-be^2)\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)} - \frac{ae\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{c}\,(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}(4bcd+8ad^2-be^2)\,x\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+x(e+dx)}}\right)+2\sqrt{d}\left(\sqrt{c}\sqrt{c+x(e+dx)}(-4ad+bx(e\right.\right.}{8\sqrt{c}d^{3/2}x\,(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(4*b*c*d + 8*a*d^2 - b*e^2)*x*ArcTanh[(e + 2*
d*x)/(2*Sqrt[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c]*Sqrt[c + x*(e
+ d*x)]*(-4*a*d + b*x*(e + 2*d*x)) + 4*a*d*e*x*ArcTanh[(Sqrt[d]*x - Sqrt[c
+ x*(e + d*x)])/Sqrt[c]])))/(8*Sqrt[c]*d^(3/2)*x*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{a\sqrt{d\,x^2+ex+c}\,\sqrt{(b\,x^2+a)^2}}{x(b\,x^2+a)}+\frac{\left(\sqrt{d}\,a\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)+\frac{bc\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{2\sqrt{d}}-\frac{ae\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)}{2\sqrt{c}}+\right.}{b\,x^2+a}$
default	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(8d^{\frac{7}{2}}\sqrt{d\,x^2+ex+c}\,a\,x^2-4d^{\frac{5}{2}}\sqrt{c}\,\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)aex+4d^{\frac{5}{2}}\sqrt{d\,x^2+ex+c}\,bc\,x^2-8d^{\frac{5}{2}}\left(d\,x^2+ex+c\right)^{\frac{3}{2}}a+8d^{\frac{5}{2}}\right.}{}$

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*(d*x^2+e*x+c)^(1/2)/x*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+(d^(1/2)*a*ln((1/2*e
+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))+1/2*b*c*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*
x+c)^(1/2))/d^(1/2)-1/2*a*e/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/
```

```
2))/x)+1/2*b*x*(d*x^2+e*x+c)^(1/2)+1/4*b/d*e*(d*x^2+e*x+c)^(1/2)-1/8*b/d^(3
/2)*e^2*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2)))*((b*x^2+a)^2)^(1/2)/(b
*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$$
$$= \left[\frac{4\,a\sqrt{c}d^2ex\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-(4\,bc^2d+8\,acd^2-bce^2)\sqrt{d}x\log\left(8\,d^2x^2+8\,de\right.}{16\,cd^2x}\right.$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] [1/16*(4*a*sqrt(c)*d^2*e*x*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2
+ e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c^2*d + 8*a*c*d^2 - b*c
*e^2)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(
d*x^2 + e*x + c))/(c*d^2*x), 1/8*(2*a*sqrt(c)*d^2*e*x*log((8*c*e*x + (4*c*d
 + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (
4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*c*d^2*x^2 + b*c*d*
e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x), 1/16*(8*a*sqrt(-c)*d^2*e
*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x +
 c^2)) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*
x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(2*b*c*d
^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x), 1/8*(4*a*
sqrt(-c)*d^2*e*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d
*x^2 + c*e*x + c^2)) - (4*b*c^2*d + 8*a*c*d^2 - b*c*e^2)*sqrt(-d)*x*arctan(
1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2
*(2*b*c*d^2*x^2 + b*c*d*e*x - 4*a*c*d^2)*sqrt(d*x^2 + e*x + c))/(c*d^2*x)]
```

Sympy [F]

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx^2)^2}}{x^2}\,dx$$

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x**2)**2)/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima"
)
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{ae\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)\mathrm{sgn}(bx^2+a)}{\sqrt{-c}} + \frac{1}{4}\sqrt{dx^2+ex+c}\left(2\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{be\mathrm{sgn}(bx^2+a)}{d}\right) - \frac{(4\,bcd\mathrm{sgn}(bx^2+a)+8\,ad^2\mathrm{sgn}(bx^2+a)-be^2\mathrm{sgn}(bx^2+a))\log\left(\left|2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}+e\right|\right)}{8\,d^{\frac{3}{2}}} + \frac{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)ae\mathrm{sgn}(bx^2+a)+2\,ac\sqrt{d}\mathrm{sgn}(bx^2+a)}{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^2-c}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] a*e*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))*sgn(b*x^2 + a)/sq
rt(-c) + 1/4*sqrt(d*x^2 + e*x + c)*(2*b*x*sgn(b*x^2 + a) + b*e*sgn(b*x^2 +
a)/d) - 1/8*(4*b*c*d*sgn(b*x^2 + a) + 8*a*d^2*sgn(b*x^2 + a) - b*e^2*sgn(b*
x^2 + a))*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(3/
2) + ((sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*e*sgn(b*x^2 + a) + 2*a*c*sqrt(d
)*sgn(b*x^2 + a))/((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx=\int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^2}\,dx$$

```
[In] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)
[Out] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)
```

3.41 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 288

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\,(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)} + \frac{be\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{d}\,(a+bx^2)} - \frac{(8bc^2+4acd-ae^2)\sqrt{a^2+2abx^2+b^2x^4}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{8c^{3/2}\,(a+bx^2)}$$

```
[Out] -1/2*a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^2/(b*x^2+a)-1/8*(4*a*c*d
-a*e^2+8*b*c^2)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x^2+
a)^2)^(1/2)/c^(3/2)/(b*x^2+a)+1/2*b*e*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+
e*x+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/d^(1/2)+1/4*(a*e+2*(a*d+2*b*c)*
x)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6876, 1664, 826, 857, 635, 212, 738}

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= -\frac{\sqrt{a^2+2abx^2+b^2x^4}(4acd-ae^2+8bc^2)\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}\,(a+bx^2)}$$
$$+\frac{be\sqrt{a^2+2abx^2+b^2x^4}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}\,(a+bx^2)}$$
$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{2cx^2\,(a+bx^2)}$$
$$+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}(2x(ad+2bc)+ae)}{4cx\,(a+bx^2)}$$

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((a*e + 2*(2*b*c + a*d)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(4*c*x*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])/(2*c*x^2*(a + b*x^2)) + (b*e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*
ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x
^2)) - ((8*b*c^2 + 4*a*c*d - a*e^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh
[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 6876

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(2ab+2b^2x^2)\sqrt{c+ex+dx^2}}{x^3}\,dx}{2ab+2b^2x^2}$$

$$= -\frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(abe-2b(2bc+ad)x)\sqrt{c+ex+dx^2}}{x^2}\,dx}{2c\,(2ab+2b^2x^2)}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{b(8bc^2+4acd-ae^2)+4b^2cex}{x\sqrt{c+ex+dx^2}}\,dx}{4c\left(2ab+2b^2x^2\right)}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(b^2e\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x^2} + \frac{\left(b\left(8bc^2+4acd-ae^2\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{4c\left(2ab+2b^2x^2\right)}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(2b^2e\sqrt{a^2+2abx^2+b^2x^4}\right)\mathrm{Subst}\left(\int \frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x^2} - \frac{\left(b\left(8bc^2+4acd-ae^2\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{2c\left(2ab+2b^2x^2\right)}$$

$$= \frac{(ae+2(2bc+ad)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{4cx\left(a+bx^2\right)} - \frac{a\left(c+ex+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{be\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{d}\left(a+bx^2\right)} - \frac{\left(8bc^2+4acd-ae^2\right)\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{8c^{3/2}\left(a+bx^2\right)}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = -\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}(8bc^2+4acd-ae^2)\,x^2\text{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)+\sqrt{c}\left(\sqrt{d}(2ac+aex-4bcx^2)\sqrt{c+\cdots}\right.\right.}{4c^{3/2}\sqrt{d}x^2\,(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*(8*b*c^2 + 4*a*c*d - a*e^2)*x^2*ArcTanh[
(-(Sqrt[d]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]] + Sqrt[c]*(Sqrt[d]*(2*a*c +
 a*e*x - 4*b*c*x^2)*Sqrt[c + x*(e + d*x)] + 2*b*c*e*x^2*Log[e + 2*d*x - 2*S
qrt[d]*Sqrt[c + x*(e + d*x)]])))/(c^(3/2)*Sqrt[d]*x^2*(a + b*x^2))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{a\sqrt{d\,x^2+ex+c}\,(ex+2c)\sqrt{(b\,x^2+a)^2}}{4x^2c(b\,x^2+a)}+\frac{\left(\frac{8ebc\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{\sqrt{d}}+8bcd\left(\frac{\sqrt{d\,x^2+ex+c}}{d}-\frac{e\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{2d^{\frac{3}{2}}}\right)-\frac{(4acd\cdots}{\cdots}\right)}{8c(b\,x^2+a)}$
default	$\frac{\sqrt{(b\,x^2+a)^2}\left(-4d^{\frac{5}{2}}c^{\frac{3}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)a\,x^2-8d^{\frac{3}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)b\,x^2-2d^{\frac{5}{2}}\sqrt{d\,x^2+ex+c}\,ae\,x^3+4d^{\frac{5}{2}}\sqrt{\cdots}\right.}{\cdots}$

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a*(d*x^2+e*x+c)^(1/2)*(e*x+2*c)/x^2/c*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/
8/c*(8*e*b*c*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))/d^(1/2)+8*b*c*d*(1
/d*(d*x^2+e*x+c)^(1/2)-1/2*e/d^(3/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(
1/2)))-(4*a*c*d-a*e^2+8*b*c^2)/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^
(1/2))/x))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 749, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= \left[\frac{4\,bc^2\sqrt{d}ex^2\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)-(8\,bc^2d+4\,acd^2-ade^2)\sqrt{}}{16\,c^2dx^2}\right.$$
$$-\frac{8\,bc^2\sqrt{-d}ex^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)+(8\,bc^2d+4\,acd^2-ade^2)\sqrt{c}x^2\log\left(\frac{8\,cex+(4\,cd+e^2)x^2+4\,\sqrt{dx^2+e}}{x^2}\right.}{16\,c^2dx^2}$$
$$-\frac{4\,bc^2\sqrt{-d}ex^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-(8\,bc^2d+4\,acd^2-ade^2)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{}}{2\,(cdx^2+cex+c^2)}\right.}{8\,c^2dx^2}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas"
)

[Out] [1/16*(4*b*c^2*sqrt(d)*e*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2)*
sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x
 + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*s
qrt(d*x^2 + e*x + c))/(c^2*d*x^2), -1/16*(8*b*c^2*sqrt(-d)*e*x^2*arctan(1/2
*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (8*b
*c^2*d + 4*a*c*d^2 - a*d*e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2
+ 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(4*b*c^2*d*
x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2 + e*x + c))/(c^2*d*x^2), 1/8*(2*b*c
^2*sqrt(d)*e*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x +
 e)*sqrt(d) + 4*c*d + e^2) + (8*b*c^2*d + 4*a*c*d^2 - a*d*e^2)*sqrt(-c)*x^2
*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c
^2)) + 2*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2 + e*x + c))/(c^
2*d*x^2), -1/8*(4*b*c^2*sqrt(-d)*e*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*
d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (8*b*c^2*d + 4*a*c*d^2 - a*d*e
^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d
*x^2 + c*e*x + c^2)) - 2*(4*b*c^2*d*x^2 - a*c*d*e*x - 2*a*c^2*d)*sqrt(d*x^2
 + e*x + c))/(c^2*d*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e^2-4*c*d>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$$
$$= -\frac{be\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}-e\right|\right)\operatorname{sgn}(bx^2+a)}{2\sqrt{d}}$$
$$+\sqrt{dx^2+ex+c}\,b\operatorname{sgn}\left(bx^2+a\right)$$
$$+\frac{\left(8\,bc^2\operatorname{sgn}(bx^2+a)+4\,acd\operatorname{sgn}(bx^2+a)-ae^2\operatorname{sgn}(bx^2+a)\right)\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}c}$$
$$+\frac{4\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3acd\operatorname{sgn}(bx^2+a)+\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3ae^2\operatorname{sgn}(bx^2+a)+8\left(\sqrt{d}x-\right.}{4\left(\left(\sqrt{\right.}\right.}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*e*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) - e))*sgn(b
*x^2 + a)/sqrt(d) + sqrt(d*x^2 + e*x + c)*b*sgn(b*x^2 + a) + 1/4*(8*b*c^2*s
gn(b*x^2 + a) + 4*a*c*d*sgn(b*x^2 + a) - a*e^2*sgn(b*x^2 + a))*arctan(-(sqr
t(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(sqrt(-c)*c) + 1/4*(4*(sqrt(d)*x
- sqrt(d*x^2 + e*x + c))^3*a*c*d*sgn(b*x^2 + a) + (sqrt(d)*x - sqrt(d*x^2 +
 e*x + c))^3*a*e^2*sgn(b*x^2 + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2
*a*c*sqrt(d)*e*sgn(b*x^2 + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c^2
*d*sgn(b*x^2 + a) + (sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c*e^2*sgn(b*x^2 +
 a))/(((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx=\int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^3}\,dx$$

```
[In] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)
[Out] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)
```

3.42 $\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4} dx$

Optimal result

Integrand size = 40, antiderivative size = 294

$$\begin{aligned} &\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx \\ &= \frac{(2ace-(8bc^2-ae^2)\,x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2\,(a+bx^2)} \\ &\quad - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3\,(a+bx^2)} \\ &\quad + \frac{b\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{a+bx^2} \\ &\quad - \frac{e(8bc^2-a(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{16c^{5/2}\,(a+bx^2)} \end{aligned}$$

```
[Out] -1/3*a*(d*x^2+e*x+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^3/(b*x^2+a)-1/16*e*(8*b*
c^2-a*(4*c*d-e^2))*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x
^2+a)^2)^(1/2)/c^(5/2)/(b*x^2+a)+b*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x
+c)^(1/2))*d^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/8*(2*a*c*e-(-a*e^2+8*b*c
^2)*x)*(d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c^2/x^2/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {6876, 1664, 824, 857, 635, 212, 738}

$$\begin{aligned}&\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx\\ &= -\frac{e\sqrt{a^2+2abx^2+b^2x^4}(8bc^2-a(4cd-e^2))\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{16c^{5/2}\,(a+bx^2)}\\ &+\frac{b\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{a+bx^2}\\ &+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2+ex}(2ace-x(8bc^2-ae^2))}{8c^2x^2\,(a+bx^2)}\\ &-\frac{a\sqrt{a^2+2abx^2+b^2x^4}(c+dx^2+ex)^{3/2}}{3cx^3\,(a+bx^2)}\end{aligned}$$

```
[In] Int[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^4,x]
```

```
[Out] ((2*a*c*e - (8*b*c^2 - a*e^2)*x)*Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(8*c^2*x^2*(a + b*x^2)) - (a*(c + e*x + d*x^2)^(3/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(3*c*x^3*(a + b*x^2)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a
+ b*x^2) - (e*(8*b*c^2 - a*(4*c*d - e^2))*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*A
rcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2])])/(16*c^(5/2)*(a + b*x
^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = Polynomia
lRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^
(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b
*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m +
1)*(c*d^2 - b*d*e + a*e^2)*Q + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq, x]
&& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Rule 6876

```
Int[(u_)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> D
ist[Sqrt[a + b*x^n + c*x^(2*n)]/((4*c)^(p - 1/2)*(b + 2*c*x^n)), Int[u*(b +
 2*c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ
[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(2ab+2b^2x^2)\sqrt{c+ex+dx^2}}{x^4}\,dx}{2ab+2b^2x^2}$$

$$= -\frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3(a+bx^2)} - \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(3abe-6b^2cx)\sqrt{c+ex+dx^2}}{x^3}\,dx}{3c(2ab+2b^2x^2)}$$

$$= \frac{(2ace-(8bc^2-ae^2)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3(a+bx^2)} + \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{\frac{3}{2}be(8bc^2-a(4cd-e^2))+24b^2c^2dx}{x\sqrt{c+ex+dx^2}}\,dx}{12c^2(2ab+2b^2x^2)}$$

$$= \frac{(2ace-(8bc^2-ae^2)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3(a+bx^2)} + \frac{\left(2b^2d\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x^2} + \frac{\left(be(8bc^2-a(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\right)\int \frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{8c^2(2ab+2b^2x^2)}$$

$$= \frac{(2ace-(8bc^2-ae^2)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3(a+bx^2)} + \frac{\left(4b^2d\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x^2} - \frac{\left(be(8bc^2-a(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{4c^2(2ab+2b^2x^2)}$$

$$= \frac{(2ace-(8bc^2-ae^2)x)\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8c^2x^2(a+bx^2)} - \frac{a(c+ex+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3cx^3(a+bx^2)} + \frac{b\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{a+bx^2} - \frac{e(8bc^2-a(4cd-e^2))\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{16c^{5/2}(a+bx^2)}$$

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$$
$$= \frac{\sqrt{(a+bx^2)^2}\left(-3e(8bc^2+a(-4cd+e^2))\,x^3\text{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)-\sqrt{c}\left(\sqrt{c+x(e+dx)}(24bc^2x^2+\right.\right.}{24c^{5/2}x^3\,(a+bx^2)}$$

```
[In] Integrate[(Sqrt[c + e*x + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*e*(8*b*c^2 + a*(-4*c*d + e^2))*x^3*ArcTanh[(-(Sqrt
[d]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]] - Sqrt[c]*(Sqrt[c + x*(e + d*x)]*(
24*b*c^2*x^2 + a*(8*c^2 - 3*e^2*x^2 + 2*c*x*(e + 4*d*x))) + 24*b*c^2*Sqrt[d
]*x^3*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]])))/(24*c^(5/2)*x^3*(
a + b*x^2))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{\sqrt{dx^2+ex+c}\,(8acdx^2-3ae^2x^2+24bc^2x^2+2acex+8ac^2)\sqrt{(bx^2+a)^2}}{24x^3c^2(bx^2+a)}-\frac{\left(-16bc^2\sqrt{d}\,\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{dx^2+ex+c}\right)-\frac{e\left(4acd-ae^2-\right.}{}\right.}{16c^2(bx^2}$
default	$\frac{\sqrt{(bx^2+a)^2}\left(12d^{\frac{5}{2}}c^{\frac{3}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)aex^3-24d^{\frac{3}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\sqrt{dx^2+ex+c}}{x}\right)bex^3+6d^{\frac{5}{2}}\sqrt{dx^2+ex+c}\,ae^2x^4+48\right.}{}$

```
[In] int((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(d*x^2+e*x+c)^(1/2)*(8*a*c*d*x^2-3*a*e^2*x^2+24*b*c^2*x^2+2*a*c*e*x+8
*a*c^2)/x^3/c^2*((b*x^2+a)^2)^(1/2)/(b*x^2+a)-1/16/c^2*(-16*b*c^2*d^(1/2)*l
n((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))-e*(4*a*c*d-a*e^2-8*b*c^2)/c^(1/2
)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x))*((b*x^2+a)^2)^(1/2)/(b*x^2
+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.59 (sec) , antiderivative size = 791, normalized size of antiderivative = 2.69

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$$
$$= \left[\frac{48\,bc^3\sqrt{d}x^3\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+3\,(ae^3+4\,(2\,bc^2-acd)e)\cdots}{}\right.$$
$$-\frac{96\,bc^3\sqrt{-d}x^3\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-3\,(ae^3+4\,(2\,bc^2-acd)e)\sqrt{c}x^3\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+\cdots}}{x^2}\cdots\right.}{96\,c^3x^3}$$
$$-\frac{48\,bc^3\sqrt{-d}x^3\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-3\,(ae^3+4\,(2\,bc^2-acd)e)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{dx^2+ex+c}(ex+2\,c)\cdots}{2\,(cdx^2+cex+c^2)}\cdots\right.}{48\,c^3x^3}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="fricas"
)

[Out] [1/96*(48*b*c^3*sqrt(d)*x^3*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x +
c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*s
qrt(c)*x^3*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x
+ 2*c)*sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c
^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3), -1/96*(96*b*c^3*sq
rt(-d)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 +
 d*e*x + c*d)) - 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt(c)*x^3*log((8*c*e*x
 + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)
/x^2) + 4*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2)*x^2)*
sqrt(d*x^2 + e*x + c))/(c^3*x^3), 1/48*(24*b*c^3*sqrt(d)*x^3*log(8*d^2*x^2
+ 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 3*
(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt(-c)*x^3*arctan(1/2*sqrt(d*x^2 + e*x +
c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2*e*x + 8*a*c^3
 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3)
, -1/48*(48*b*c^3*sqrt(-d)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)
*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - 3*(a*e^3 + 4*(2*b*c^2 - a*c*d)*e)*sqrt
(-c)*x^3*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c
*e*x + c^2)) + 2*(2*a*c^2*e*x + 8*a*c^3 + (24*b*c^3 + 8*a*c^2*d - 3*a*c*e^2
)*x^2)*sqrt(d*x^2 + e*x + c))/(c^3*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**2+e*x+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e^2-4*c*d>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(224) = 448$.

Time = 0.35 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$$
$$= -b\sqrt{d}\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}-e\right|\right)\operatorname{sgn}\left(bx^2+a\right)$$
$$+\frac{\left(8\,bc^2e\operatorname{sgn}(bx^2+a)-4\,acde\operatorname{sgn}(bx^2+a)+ae^3\operatorname{sgn}(bx^2+a)\right)\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{8\sqrt{-c}c^2}$$
$$+\frac{24\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^5bc^2e\operatorname{sgn}(bx^2+a)+12\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^5acde\operatorname{sgn}(bx^2+a)-3\left(\sqrt{\right.}\right.}{}$$

```
[In] integrate((d*x^2+e*x+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -b*sqrt(d)*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) - e))*sgn
(b*x^2 + a) + 1/8*(8*b*c^2*e*sgn(b*x^2 + a) - 4*a*c*d*e*sgn(b*x^2 + a) + a*
```

```
e^3*sgn(b*x^2 + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(
sqrt(-c)*c^2) + 1/24*(24*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*b*c^2*e*sgn(
b*x^2 + a) + 12*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*a*c*d*e*sgn(b*x^2 + a
) - 3*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^5*a*e^3*sgn(b*x^2 + a) + 48*(sqrt
(d)*x - sqrt(d*x^2 + e*x + c))^4*b*c^3*sqrt(d)*sgn(b*x^2 + a) + 48*(sqrt(d)
*x - sqrt(d*x^2 + e*x + c))^4*a*c^2*d^(3/2)*sgn(b*x^2 + a) - 48*(sqrt(d)*x
- sqrt(d*x^2 + e*x + c))^3*b*c^3*e*sgn(b*x^2 + a) + 48*(sqrt(d)*x - sqrt(d*
x^2 + e*x + c))^3*a*c^2*d*e*sgn(b*x^2 + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*
x + c))^3*a*c*e^3*sgn(b*x^2 + a) - 96*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2
*b*c^4*sqrt(d)*sgn(b*x^2 + a) + 48*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2*a*
c^2*sqrt(d)*e^2*sgn(b*x^2 + a) + 24*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*b*c
^4*e*sgn(b*x^2 + a) + 36*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c^3*d*e*sgn(
b*x^2 + a) + 3*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c^2*e^3*sgn(b*x^2 + a)
 + 48*b*c^5*sqrt(d)*sgn(b*x^2 + a) + 16*a*c^4*d^(3/2)*sgn(b*x^2 + a))/(((sq
rt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)^3*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+ex+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx=\int \frac{\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^4}\,dx$$

```
[In] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^4,x)
[Out] int((((a + b*x^2)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^4, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```